Odd Generalized Exponential Chen Distributions with Applications

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ملخص البحث

في كثير من الأحيان لا يمكن معالجة البيانات من خــلال التوزيعــات التقليدية لذا كان لزاماً إيجاد توزيعات جديدة تصلح لمعالجة تلــك البيانــات، الأمر الذي أوجد الحاجة إلى استخدام التوزيعات المعممة حيــث اعتمــدت دراسات توليد التوزيعات الجديدة من توزيعات موجودة بالفعل على أسلوبين **الأول:** وهو إضافة معلمة جديدة إلى توزيع موجود بالفعل **والأسلوب الثاني:** هو الدمج بين توزيعين أو أكثر من التوزيعات الموجودة بالفعل وذلك لإيجاد توزيع جديد يحصل خصائص جيدة وسيتم دراسة الأسلوب الثاني من خلال توزيعات العائلة الآسية المعممة المرجحة وذلك بالتطبيق على توزيع تــشن ومن ثم دراسة الخصائص الإحصائية والرياضية للتوزيــع المقتــرح وفــي النهاية دراسة تقدير المعلم بطريقة "دالة الإمكان الأكبر" ولبيان مدى ملائمة التوزيع المقترح تم التطبيق على مجموعة بيانات حقيقية.

Abstract

The modeling and analysis of lifetimes is an important aspect of statistical work in a wide variety of scientific and technological fields. The study suggested for the first time, the called Odd generalized-Exponential Chen (OGECD) distribution. The new suggested distribution can have a decreasing and upside-down bathtub failure rate function depending on the value of its parameters; it's including some special sub-model like generalized Pareto distribution and its exponentiated. Some structural properties of the suggested distribution are studied including explicit expressions for the moments. The density function of the order statistics and their moments are obtained. Maximum likelihood is used for estimating the distribution parameters and the observed information matrix is derived. The information matrix is easily numerically determined. Monte Carlo simulations and the application of two real data sets are performed to illustrate the potentiality of this distribution.

Keywords and Phrases**:** Odd Generalized-Exponential, Chen Distribution, Hazard function, Moment, Maximum likelihood estimation**.**

1-INTRODUCTION

When modeling monotonic hazard rates, the exponential, gamma, lognormal, and Weibull distributions may be initial choices. However, these distributions have several limitations.

First, none of them exhibit bathtub shapes for their hazard rate functions. These distributions exhibit only monotonically increasing, decreasing, or constant hazard rates. The most realistic hazard rate is bathtub-shaped. This occurs in most real-life systems. For instance, such shapes occur when the population is divided into several subpopulations having early failures, wear out failures, and more or less constant failures. Therefore, a perfect bathtub consists of two change points and a constant part enclosed within the change points. Usefulness of bathtub shape is well recognized in several fields. Many parametric probability distributions have been introduced to analyze real datasets with bathtub failure rates. Chen (2000) proposed a new two-parameter lifetime distribution with bathtub-shaped or increasing failure rate (IFR) function. Let X be a non-negative random variable with Chen's distribution, then its corresponding cumulative and probability distribution functions (c.d.f) and (p.d.f) is given

$$F(x;\lambda,\beta) = 1-\exp\{\lambda\left(1-e^{x^{\beta}}\right)\} \qquad (1)$$

$$f(x;\lambda,\beta) = \lambda\,\beta\,x^{\beta-1}\exp\{\lambda\left(1-e^{x^{\beta}}\right)+x^{\beta}\} \qquad x>0 \qquad (2)$$

where λ > 0 and β > 0 are shapes parameters. The new two-parameter distribution has some useful properties compared with other well-known models. Xie et al. (2002) extended the

Chen’s distribution adding other parameter and named it the extended-Weibull distribution, due to relation to the Weibull distribution. Pappas et al. (2012) proposed a four-parameter modified Weibull extension distribution using the Marshall and Olkin (1997) technique. Therefore, one of its particular cases could be named as Marshall-Olkin extended Chen’s distribution.

Generated families of continuous distributions are recent development which provide great flexibility in modelling real data. These families are obtained by introducing one or more additional shape parameter(s) to the baseline distribution. Some of the generated families are listed as follows; the beta- genertaed (B-G) (Eugene et al. 2002) (Jones, M.C 2004), gamma-G (type 1) (Zografos and Balakrishnan 2009), Kumaraswamy-G (Cordeiro and Castro 2011), McDonald-G (Alexander et al. 2012), gamma-G (type 2) (Risti ́c and Balakrishnan 2012), transformed-transformer-G (Alzaatreh et al. 2013), Weibull-G (Bourguignon et al. 2014), *odd generalized exponential-G* (OGE-G) (Tahir et al. 2015), Kumaraswamy Weibull-G (Hassan and Elgarhy 2016), among others. Our interest here, with the OGE-G family which is flexible because of the hazard rate shapes: increasing, decreasing, J, reversed-J, bathtub and upside-down bathtub. The cdf and pdf of the OGE-G are defined as follows

$$F(x) = F(x;\, \alpha, \lambda, \xi) = \left(1 - e^{-\delta \frac{G(x;\xi)}{\bar{G}(x;\xi)}}\right)^{\alpha} \qquad (3)$$

$$f(x) = f(x;\, \alpha, \lambda, \xi) = \frac{\delta\, \alpha\, g(x;\xi)}{\bar{G}(x;\xi)^{2}}\, e^{-\lambda \frac{G(x;\xi)}{\bar{G}(x;\xi)}} \left(1 - e^{-\delta \frac{G(x;\xi)}{\bar{G}(x;\xi)}}\right)^{\alpha-1} \qquad (4)$$

where $g(x;\, \xi)$ is the baseline pdf. We can omit the dependence on the vector of parameters ξ and write simply $G(x) = G(x;\, \xi)$. Equation 4 will be most tractable when the cdf $G(x)$ and pdf $g(x)$ have explicit expressions. Hereafter, a random variable X with density function (4) is denoted by $X \sim OGE(\alpha, \delta, \xi)$. The main motivations for using the OGE family are to make the kurtosis more flexible (compared to the baseline model) and possible to construct heavy-tailed distributions that are not long-tailed for modeling real data.

The study offer a physical interpretation of X when α is an integer. Consider a system formed by α independent components following the *odd exponential-G class* (Bourguignon et al. 2014) given by

$$H(x;\ \lambda,\xi) = 1 - e^{-\delta \frac{G(x;\,\xi)}{\bar{G}(x;\,\xi)}}$$

Suppose the system fails if all α components fail and let X denote the lifetime of the entire system. Then, the cdf of X is $F(x;\ \alpha,\delta,\xi) = H(x;\ \delta,\xi)^{\alpha}$, which is identical to (1).

The hrf of X is given by

$$h(x) = h(x;\ \alpha,\delta,\xi) = \frac{\delta\ \alpha\ e^{-\delta \frac{G(x;\,\xi)}{\bar{G}(x;\,\xi)}} \left(1 - e^{-\delta \frac{G(x;\,\xi)}{\bar{G}(x;\,\xi)}}\right)^{\alpha-1}}{\bar{G}(x;\xi)^2 \left\{1 - \left(1 - e^{-\delta \frac{G(x;\,\xi)}{\bar{G}(x;\,\xi)}}\right)^{\alpha}\right\}}$$

(5)

To increase the flexibility for modeling purposes it will be useful to consider further alternatives to PF (under study in this paper) distribution. Our purpose is to provide a new four-parameter model, named as odd generalized exponential Chen

Distribution (OGECD) using the OGE-G family. The suggested model is quite flexible in terms of hazard rate could be increasing, decreasing, U and J-shaped. Also, paper show its flexibility on the basis of three real life data.

This paper is outlined as follows. In section 2, we define the OGECD distribution and provide expansions for its cumulative and density functions. A range of mathematical properties of this distribution is considered in sections 3. Maximum likelihood estimation is performed and the observed information matrix is determined in section 4. In section 5, we provide application to several real data sets to illustrate the potentiality of this distribution. Finally, some conclusions are addressed in section 6.

2- THE OGECD DISTRIBUTION

In this section, the study introduce the new suggested odd generalized exponential Chen distribution. The pdf, cdf, reliability function, hrf, reversed-hazard rate function and cumulative hazard rate function of the OGECD distribution are derived

The probability distribution function is:

$$f(x) = f(x;\alpha,\lambda,\xi) = \frac{\delta\,\alpha\,\lambda\,\beta\,x^{\beta-1}\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)+x^{\beta}\right\}}{\left[1-\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}\right]^{2}}\,e^{-\delta\frac{1-\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}}{\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}}}$$

$$\left(1-e^{-\delta\frac{1-\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}}{\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}}}\right)^{\alpha-1} \quad (6)$$

where $\underline{\xi} = (\lambda,\beta)$ and (α,δ), are non-negative shape Parameters. The corresponding cdf and Hazard Rate Function are

If $F(x;\underline{\theta})$ is the Chen cumulative distribution (1) with Parameter $\underline{\theta} = (\lambda,\beta)$ then equation (3) yields the OGEC cumulative distribution

$$F(x) = F(x;\,\alpha,\delta,\lambda,\beta) = \left(1-e^{-\delta\frac{1-\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}}{\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}}}\right)^{\alpha} \quad x>0 \quad (7)$$

and

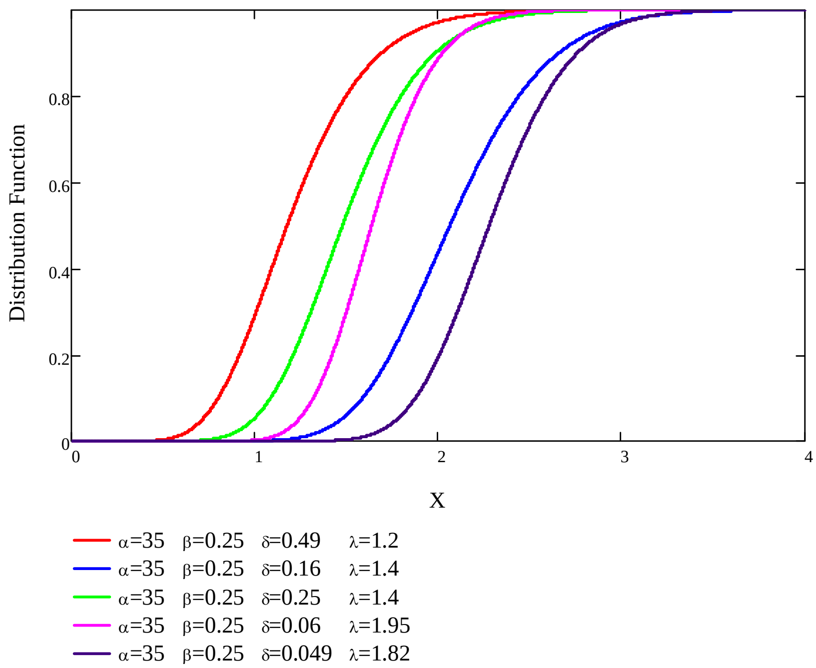
$$S(x;\,\alpha,\delta,\lambda,\beta) = 1-F(x;\,\alpha,\delta,\lambda,\beta) = 1-\left(1-e^{-\delta\frac{1-\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}}{\exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\}}}\right)^{\alpha}$$

$$H(x;\underline{\xi}) = \frac{f(x;\underline{\xi})}{S(x;\underline{\xi})} = \frac{\dfrac{\delta\,\alpha\,\lambda\,\beta\, x^{\beta-1}\exp\{\lambda(1-e^{x^\beta})+x^\beta\}}{[1-\exp\{\lambda(1-e^{x^\beta})\}]^2}\, e^{-\delta\frac{1-\exp\{\lambda(1-e^{x^\beta})\}}{\exp\{\lambda(1-e^{x^\beta})\}}}}{\left[1-\left(1-e^{-\delta\frac{1-\exp\{\lambda(1-e^{x^\beta})\}}{\exp\{\lambda(1-e^{x^\beta})\}}}\right)^{\alpha}\right]\left(1-e^{-\delta\frac{1-\exp\{\lambda(1-e^{x^\beta})\}}{\exp\{\lambda(1-e^{x^\beta})\}}}\right)^{-(\alpha-1)}}$$

Respectively

Some plots of the cdf and pdf of OGECD distribution for some selected parameter values. Figure 1 indicates that the cumulative and densities of the OGECD take different shapes.

(a)


Distribution Function
0
0.2
0.4
0.6
0.8
0
1
2
3
4
X
α=35 β=0.25 δ=0.49 λ=1.2
α=35 β=0.25 δ=0.16 λ=1.4
α=35 β=0.25 δ=0.25 λ=1.4
α=35 β=0.25 δ=0.06 λ=1.95
α=35 β=0.25 δ=0.049 λ=1.82

(b)

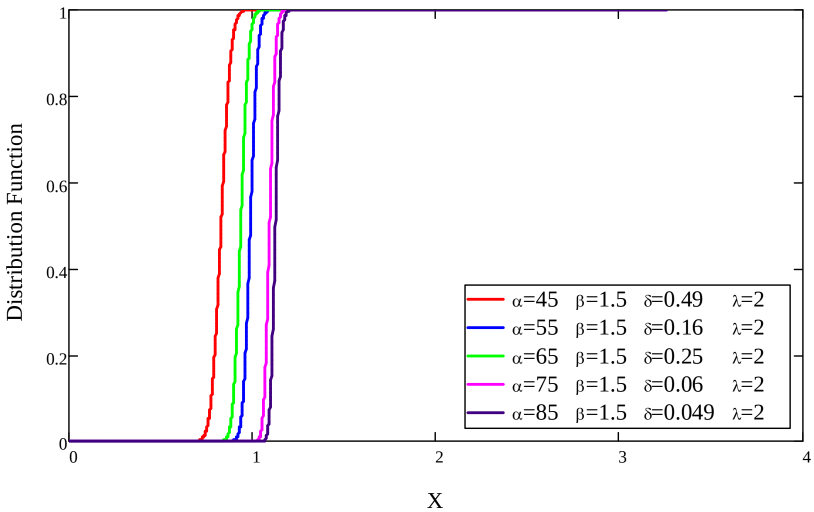

Distribution Function
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
X
α=45 β=1.5 δ=0.49 λ=2
α=55 β=1.5 δ=0.16 λ=2
α=65 β=1.5 δ=0.25 λ=2
α=75 β=1.5 δ=0.06 λ=2
α=85 β=1.5 δ=0.049 λ=2

Figure 1: Plots of the OGECD distribution function for some parameter values.

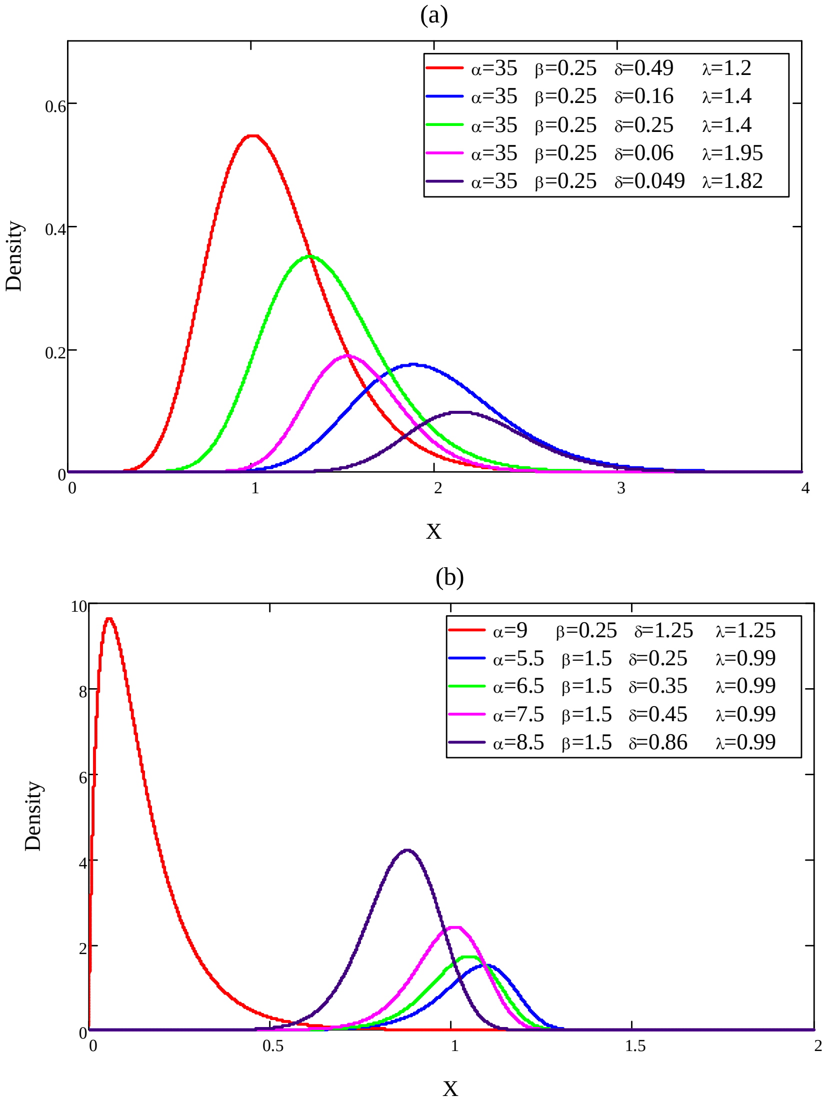


Figure 2: Plots of the OGECD density function for some parameter values.

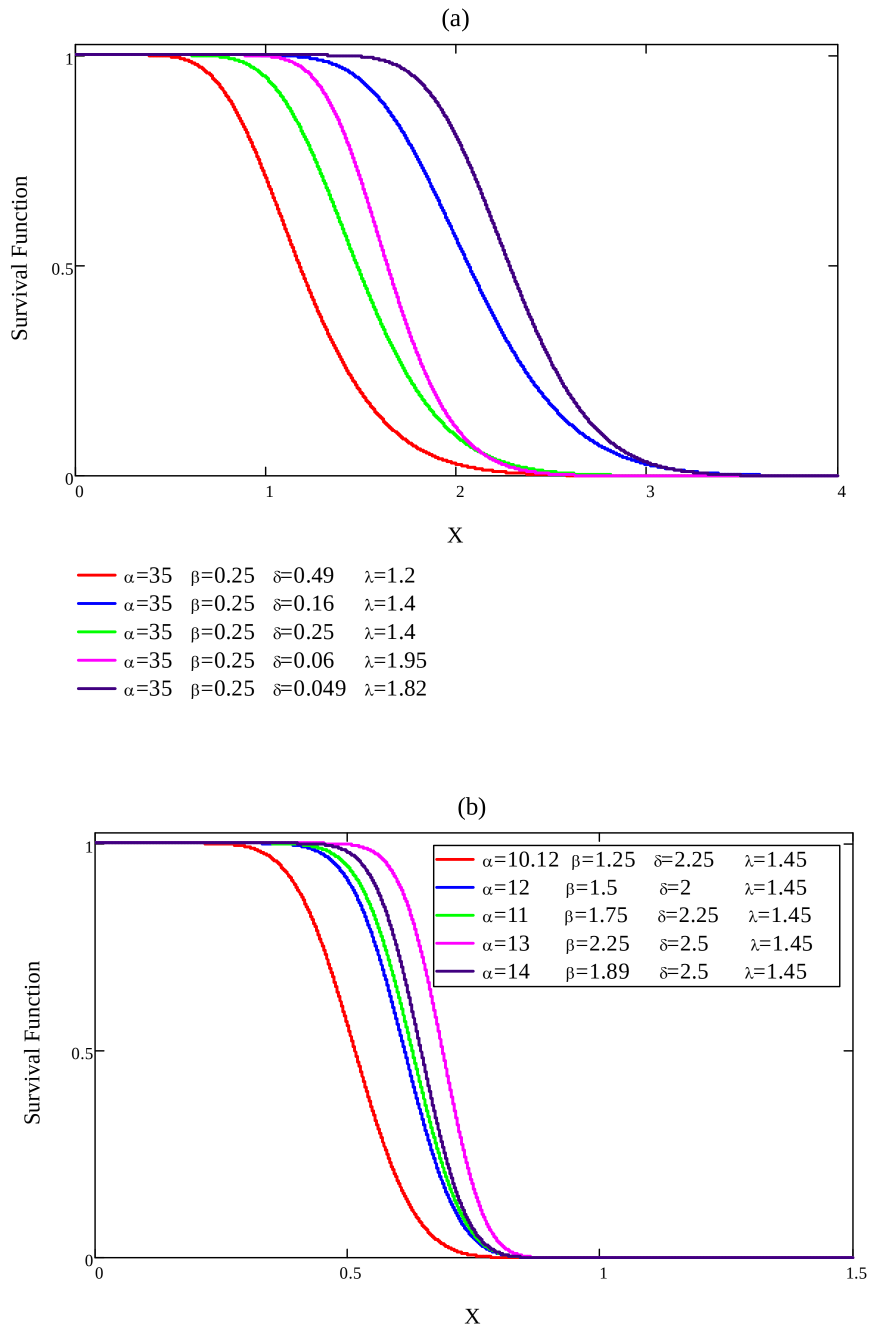


Figure 3: (a) Plots of the OGECD survival function for some values of a.

(b) Plots of the OGECD survival function for some values of b.

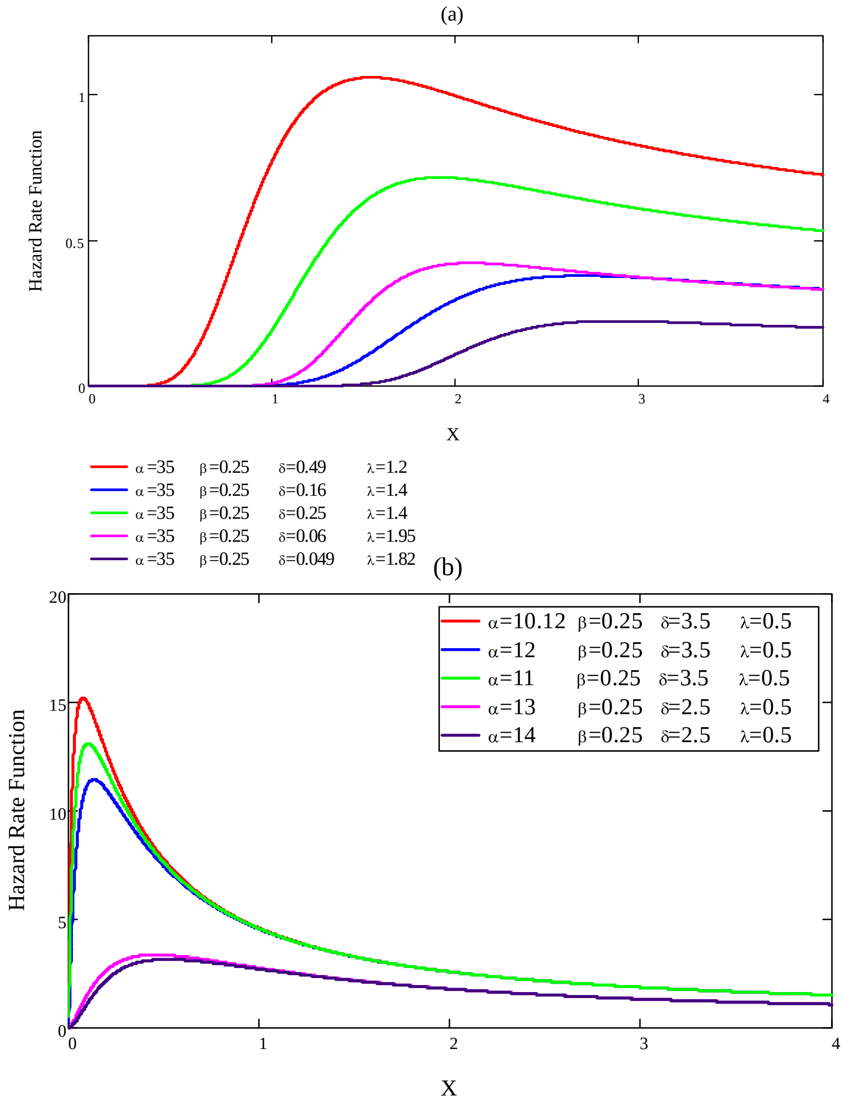


Figure 4: Plots of the OGECD hazard rate

Figure 3,4 indicates that OGECD survival and hrfs can have increasing, decreasing, J and U-shaped. This fact implies that the OGECD can be very useful for fitting data sets with various shapes.

3- Statistical and Reliability Properties

3.1 Quantile function and simulation

Here, the method for simulating from the OGEC distribution (6) is presented. The quantile function corresponding to (6) is

$$Q(u) = F^{-1}(u) = \beta\left[\left\{1-\left(1-(1-u)^{\frac{1}{\delta}}\right)^{\frac{1}{\alpha}}\right\}^{\frac{-1}{\alpha}}-1\right]^{\frac{1}{\delta}}+\lambda$$

Simulating the OGECD random variable is straightforward. Let U be a uniform variate on the unit interval $(0,1)$. Thus, by means of the inverse transformation method, we consider the random variable X given by

$$X_p = \left[\log\left(1-\frac{1}{\lambda}\log\left(\frac{1}{1-\frac{1}{\delta}\log\left(1-P^{\frac{1}{\alpha}}\right)}\right)\right)\right]^{\frac{1}{\beta}}$$

which follows (6), i.e. $X \sim OGEC\ (\alpha,\beta,\lambda,\delta)$.

3.2 Skewness and Kurtosis

The shortcomings of the classical kurtosis measure are well-known. There are many heavy tailed distributions for which this measure is infinite. So, it becomes uninformative precisely when it needs to be. Indeed, our motivation to use quantile-based measures stemmed from the non-existance of classical kurtosis for many of the OGEC distributions

The Bowley's skewness (see Kenney and Keeping 1962) is based on quartiles:

$$S_k = \frac{Q(3/4) - 2Q(1/2) + Q(1/4)}{Q(3/4) - Q(1/4)}$$

And the Moors' kurtosis (see Moors (17)) is based on octiles:

$$K_u = \frac{Q(7/8) - Q(5/8) - Q(3/8) + Q(1/8)}{Q(6/8) - Q(2/8)}$$

Where $Q(\cdot)$ represents the quantile function

4- Estimation and information matrix

In this section, the study discuss maximum likelihood estimation and inference for the OGECD distribution. Let $x_1, x_2, \dots, x_n$ be a random sample from $X \sim OGECD\left(\underline{\xi}\right)$

where $\underline{\xi} = (\alpha, \beta, \delta, \lambda)$ be the vector of the model Parameters, the log-likelihood function for $\underline{\xi}$ reduces to

$$\ell\left(\underline{\xi}\right) = n\ln\delta + n\ln\alpha + n\ln\lambda + n\ln\beta$$
$$+\sum_{i=1}^{n}\left[\lambda\left(1-e^{x_i^{\beta}}\right)+x_i^{\beta}\right] - 2\sum_{i=1}^{n}\ln\left[1-e^{\lambda\left(1-e^{x_i^{\beta}}\right)}\right]$$
$$-\delta\sum_{i=1}^{n}\frac{\left[1-e^{\lambda\left(1-e^{x_i^{\beta}}\right)}\right]}{\exp\{\lambda(1-e^{x_i^{\beta}})\}} + (\alpha-1)\sum_{i=1}^{n}\ln\left[1-e^{-\delta\frac{\left(1-e^{\lambda\left(1-e^{x_i^{\beta}}\right)}\right)}{\exp\{\lambda(1-e^{x_i^{\beta}})\}}}\right] \tag{8}$$

By setting $\omega(x_i) = \frac{1-A(x_i)}{A(x_i)}$; $A(x_i) = \exp\left\{\lambda\left(1-e^{x_i^{\beta}}\right)\right\}$

$$\frac{\partial\ell}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\log\left(1-e^{-\delta\omega(x_i)}\right)$$

$$\frac{\partial\ell}{\partial\beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\left(x_i^{\beta} - \lambda x_i^{\beta}e^{x_i^{\beta}}\right)ln\,x_i - 2\sum_{i=1}^{n}\left(\frac{A^{\backslash}(x_i)}{1-A(x_i)}\right) + \delta\sum_{i=1}^{n}\frac{A^{\backslash}(x_i)}{[A(x_i)]^2}$$
$$-(\alpha-1)\sum_{i=1}^{n}\frac{exp\left\{-\delta\frac{1-A(x_i)}{A(x_i)}\right\}\frac{A^{\backslash}(x_i)}{[A(x_i)]^2}}{1-exp\left\{-\delta\frac{1-A(x_i)}{A(x_i)}\right\}}$$

Where $A^{\backslash}(x_i) = A(x_i)\lambda x_i^{\beta}e^{x_i^{\beta}}\,ln\,x_i$

$$\frac{\partial\ell}{\partial\delta} = \frac{n}{\delta} - \sum_{i=1}^{n}\omega(x_i) + (a-1)\sum_{i=1}^{n}\frac{\omega(x_i)exp\{-\delta\omega(x_i)\}}{1-exp\{-\delta\omega(x_i)\}}$$

and

$$\frac{\partial \ell}{\partial \lambda}=\frac{n}{\lambda}+\sum_{i=1}^{n}\left(1-e^{x_i^{\beta}}\right)+2\sum_{i=1}^{n}\frac{\left(1-e^{x_i^{\beta}}\right)A(x_i)}{1-A(x_i)}$$

$$-\delta\sum_{i=1}^{n}\frac{\left(1-e^{x_i^{\beta}}\right)A_{(x_i)}^{2}+\left(1-e^{x_i^{\beta}}\right)A(x_i)[1-A(x_i)]}{A_{(x_i)}^{2}}$$

$$-(a-1)\sum_{i=1}^{n}\frac{e^{-\delta\omega(x_i)}z(x_i)}{1-A_{(x_i)}^{2}}$$

Where $z(x_i)=\delta\left[\frac{A_{(x_i)}^{2}\left(1-e^{x_i^{\beta}}\right)+\left(1-e^{x_i^{\beta}}\right)A(x_i)[1-A(x_i)]}{A_{(x_i)}^{2}}\right]$

The maximum likelihood estimates (MLEs) of the parameters are the solutions of the nonlinear equations $\nabla\ell = 0$, which are solved iteratively. The observed information matrix is

$$J_n\left(\underline{\xi}\right)=n\begin{pmatrix}\Delta_{\alpha\alpha} & \Delta_{\alpha\beta} & \Delta_{\alpha\delta} & \Delta_{\alpha\lambda}\\ \Delta_{\beta\alpha} & \Delta_{\beta\beta} & \Delta_{\beta\delta} & \Delta_{\beta\lambda}\\ \Delta_{\delta\alpha} & \Delta_{\delta\beta} & \Delta_{\delta\delta} & \Delta_{\delta\lambda}\\ \Delta_{\lambda\alpha} & \Delta_{\lambda\beta} & \Delta_{\lambda\delta} & \Delta_{\lambda\lambda}\end{pmatrix}$$

Where Δ denotes the partial second derivatives of ℓ, the above information matrix can be estimated using the parameter estimates.

5. Empirical Applications

In this section, we illustrate the usefulness of the OGECD distribution.

Real Data Applications

In this section the paper use several real data sets to compare the fits of OGECD distribution with those of comparision other models. In each case parameters are estimated via the MLE method described in Section 4 using the MATHCAD software. First describe the data sets. Then report the MLEs (and the corresponding standard errors in parentheses) of the parameters and the values of the AIC (Akaike Information Criterion), CAIC (Consistent Akaike Information Criterion) and BIC (Bayesian Information Criterion) statistics.

$$AIC = -2\ell(\hat{\theta}) + 2q, \qquad BIC = -2\ell(\hat{\theta}) + q\log(n)$$

$$CAIC = -2\ell(\hat{\theta}) + \frac{2qn}{n-q-1}$$

Where $\ell(\hat{\theta})$ denotes the log-likelihood function evaluated at the maximum likelihood estimates, q is the number of parameters, and n is the sample size. Next, shall compare the proposed OGECD distribution with several other lifetime distributions data set, Kumaraswamy Fréchet distribution KwF (Mead, et al. (2014)), the beta Fréchet (BF) (Nadarajah and

Gupta, (2004) and Souza et al., (2011)). Finally, we perform the Kolmogorov-Smirnov (K-S) statistic and $-2\ell(\hat{\theta})$ tests.

The Strengths of 1.5 Cm Glass Fibers

Here, the data set is obtained from (Faton et al. (2013)). The data are consisting of 63 of the strengths of 1.5 cm glass fibers, measured at the National Physical Laboratory, England. Unfortunately, the units of measurement are not given in the paper. The data are listed in the next table

0.55	0.93	1.25	1.36	1.49	1.52	1.58
1.61	1.64	1.68	1.73	1.81	2.00	0.74
1.04	1.27	1.39	1.49	1.53	1.59	1.61
1.66	1.68	1.76	1.82	2.01	0.77	1.11
1.28	1.42	1.5	1.54	1.6	1.62	1.66
1.69	1.76	1.84	2.24	0.81	1.13	1.29
1.48	1.5	1.55	1.61	1.62	1.66	1.7
1.77	1.84	0.84	1.24	1.3	1.48	1.51
1.55	1.61	1.63	1.67	1.7	1.78	1.89

Table 1: The Strengths of 1.5 cm Glass Fibers Data Set.

Uncensored Data "Carbon Fibers"

Here, the real data set will use here to compare the fits of the OGECD distribution and other models. Considering an uncensored data set corresponding an uncensored data set from Nichols and Padgett (2006) consisting of 100 observations on breaking stress of carbon fibers (in Gba):

Table 2: On breaking stress of carbon fibers set

1.41	0.39	2.97	1.36	0.98	2.76	4.91	3.68	1.84	1.59
1.57	1.08	2.03	1.61	2.12	1.89	2.88	2.82	2.05	3.65
1.84	1.17	3.68	2.48	0.85	1.61	2.79	4.7	2.03	1.8
2.17	1.57	5.08	2.48	1.18	3.51	2.17	1.69	1.25	4.38
3.15	2.35	2.55	2.59	2.38	2.81	2.77	2.17	2.83	1.92
3.19	2.41	0.81	5.56	1.73	1.59	2	1.22	1.12	1.71
3.39	2.43	4.2	3.33	2.55	3.31	3.31	2.85	2.56	3.56
3.7	2.74	2.73	2.5	3.6	3.11	3.27	2.87	1.47	3.11
3.75	2.81	2.95	2.97	3.39	2.96	2.53	2.67	2.93	3.22
4.42	3.68	3.19	3.22	1.69	3.28	3.09	1.87	3.15	4.9

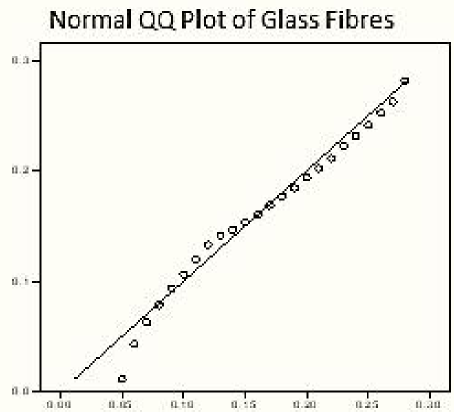
Table 3. MLEs of the model parameters, the corresponding SEs (given in parentheses) and the statistics AIC, BIC and CAIC

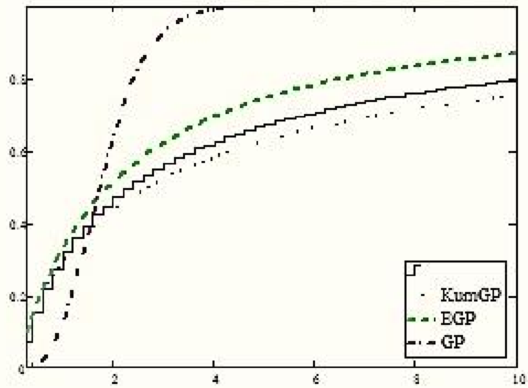
	Model	*Estimates*					*Statistic*	
		$\hat{\alpha}$	$\hat{\beta}$	β	$\hat{\lambda}$	*AIC*	*BIC*	*CAIC*
Glass Fibers	*OGECD*	0.7658	48.475	2.025	2.6843	52.296	33.012	25.349
		(0.181)	(30.634)	(10.666)	(1.031)			
	KwF	5.50397	857.34273	2.11623	0.74044	47.621	56.193	45.306
		(7.982)	(153.948)	(4.555)	(0.071)			
	BF	19.59068	30.41091	1.33081	0.6849	69.735	78.307	67.421
		(18.115)	(18.238)	(1.085)	(0.181)			
Carbon Fibers	*OGECD*	3.682	42.858	3.256	4.226	184.275	147.896	143.168
		(0.00371)	(0.1111)	(0.00001)	(0.000002)			
	KwF	6.76357	904.34345	2.90998	0.332	292.926	303.347	291.035
		(2.393)	(61.863)	(2.259)	(0.028)			
	BF	0.42934	138.06644	34.38484	0.72474	293.733	304.154	291.842
		(0.236)	(113.552)	(21.52)	(0.19)			

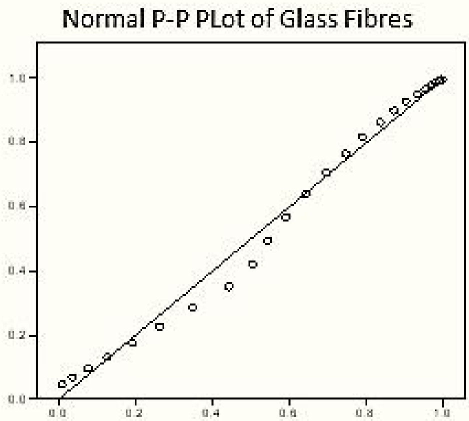
Table 4: K-S and $-2\ell(\hat{\theta})$ statistics for the chosen Real data

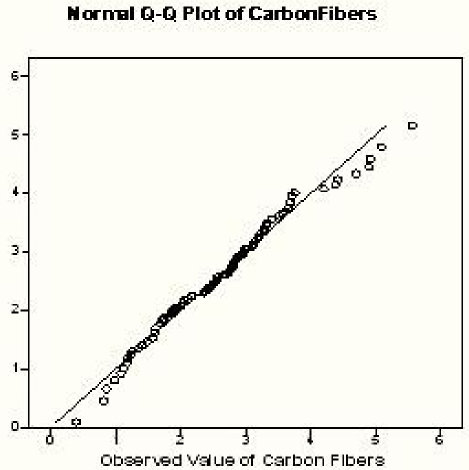
Data	Model	*OGECD*
Glass Fibers	$K-S$	**0.112**
	$-2\ell(\hat{\theta})$	**28.325**
Carbon Fibers	$K-S$	0.427
	$-2\ell(\hat{\theta})$	142.346

Since the values of the AIC, BIC and CAIC are smaller for the ***OGECD*** distribution compared with those values of the other models, the ***OGECD*** distribution seems to be a very competitive model to these data. In summary, the proposed ***OGECD*** distribution produces better fits to the data than other models.


Normal QQ Plot of Glass Fibres


KumGP
EGP
GP


Normal P-P PLot of Glass Fibres


Normal Q-Q Plot of CarbonFibers
Observed Value of Carbon Fibers

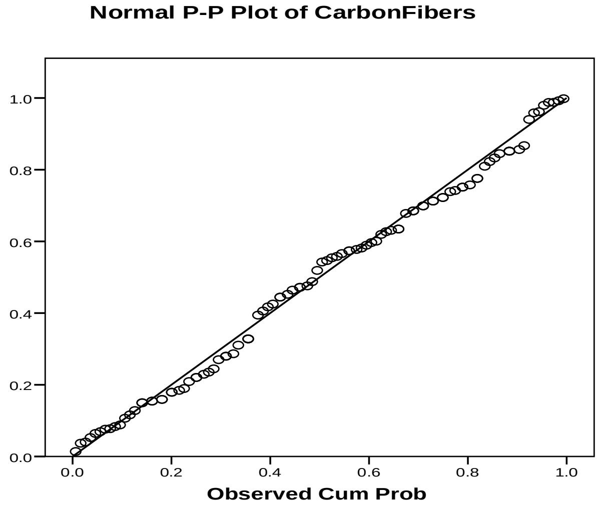


Fig. (4) The Fitted Q-Q Plots and P-P Plots for the 63 of the strengths of 1.5 cm glass fibres data set & 100 observations on breaking stress of carbon fibers and Empirical CDF.

6. CONCLUDING REMARKS

The well-known OGED distribution is extended by introducing two extra shape parameters, thus defining the Odd generalized-Exponential Chen (OGECD) distribution having a broader class of hazard rate and density functions. This is achieved by taking (1) as the baseline cumulative distribution. A detailed study on the mathematical properties of the new distribution is presented. The estimation of the model parameters is approached by maximum likelihood and the observed information matrix is obtained. An application to a real data set indicates that the fit of the new model is superior to the fits of its principal models. We hope that the proposed model may be interesting for a wider range of statistical research.

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