Optimum design plan of constant stress life testing for exponentiated Lomax distribution based on time censoring

EL Gabrey, G. A.

Statistics Department, Faculty of Commerce,

AL-Azhar University (Girls' Branch), Cairo, Egypt

Abstract

The customers' hopes are having great superiority reliable goods on period and reasonable costs. Then, researchers spend accelerated life testing that allows rapidly them reduction the examination period and keep a group of manpower, money and material sources. The major aim of this paper is to study the constant stress time censoring accelerated life tests. It is supposed that the lifetime of a test unit monitors an exponentiated lomax distribution. The estimators of maximum likelihood are developed when the parameters are unknown. The shape parameter of the lifetime distribution at constant stress stages is adopted to be a log linear model. Furthermore, confidence limits of intervals for the parameters, reliability, hazard functions are formed. Moreover, optimum constant acceleration life testing plan is studied through the asymptotic generalized variance of the maximum likelihood estimators of the parameters is minimized. Monte Carlo simulation and two real data are approved to indicate the theoretical results.

Keywords: Accelerated life tests, Maximum likelihood estimation, Confidence limits of intervals, Optimal test plan, Generalized asymptotic variance.

Introduction

By moving the life tests at upper stress stages than normal operating conditions accelerated life testing (ALT) fast produces information on the lifetime distribution of a test unit. Some main references in the region of ALT are [15], [12] and [6]

The lifetime at the scheme stress is assessed during extrapolation using a log linear model. Stresses can be used in various methods but the frequently used kinds such as step stress, constant stress and progressive stress. During physical life, many products such as, insulations, bearing, electronic components, semiconductors, and microelectronics run at a constant stress.

In constant-stress testing, a item is examined at a permanent stress level till failure occurs or the life test is ended, which comes first see, [2,3].

Several censoring criteria for ALT are spent in literature. Under time censoring, the test is accepted to happen for a pre-specified time during which the failure times for the samples are noted, see [8].

The ideal plan for constant stress ALT has got large concentration in the last three decades as the test keeps more elasticity and adjustability than ALT. [10] created a general design for constant stress ALT with multi experimental factors. [5] considered the optimal designs and statistical inference of ALT based on time constant stress for generalized logistic distribution. [4] constructed the optimal of constant stress ALT using Kumaraswamy Weibull distribution and a log-linear relationship between the stress and the shape parameter applying maximum likelihood (ML) estimation approach.

Lomax or Pareto II distribution was proposed by [11]. This distribution has wide applications such as the analysis of the business failure lifetime data, income formed a distribution with

(0,1) and two shape parameters, wealth inequality, size of cities, actuarial science, medical, biological sciences, engineering, lifetime and reliability modeling (see [9]. The exponentiated Lomax (EL) is more flexible and more fitting for real data sets, for more details see [17]. The cumulative distribution function (cdf), F(t) and the probability density function (pdf), f(t) of the EL distribution are obtained as follows:

$$F(t) = \left[1 - (1+\lambda t)^{-\beta}\right]^{\theta}, \qquad t, \beta, \theta, \lambda \geq 0,$$

(1)

$$f(t) = \beta\theta\lambda(1+\lambda t)^{-(\beta+1)}\left[1-(1+\lambda t)^{-\beta}\right]^{\theta-1}, \qquad t, \beta, \theta, \lambda \geq 0,$$

(2)

The reliability function (rf), R(t) of EL is given by

$$R(t) = 1 - \left[1 - (1+\lambda t)^{-\beta}\right]^{\theta}, \qquad t, \beta, \theta, \lambda \geq 0,$$

(3)

The hazard function (hf) of EL is given by

$$h(t) = \frac{\beta\theta\lambda(1+\lambda t)^{-(\beta+1)}\left[1-(1+\lambda t)^{-\beta}\right]^{\theta-1}}{1-\left[1-(1+\lambda t)^{-\beta}\right]^{\theta}} t, \beta, \theta, \lambda \geq 0$$

(4)

The rest of the paper is divided into five sections. Section 2 finds with the parameters estimators of the EL distribution when the accelerated life testing is constant stress under time censored data. Confidence limits of intervals for the parameters are built in Section 3. Optimal test plan under time censoring are developed in Section 4. Simulation study is shown in Section 5 and the two applications are given in Section 6.

The estimators of maximum likelihood under time censoring.

[1] presented that the log linear function of stress is just a re-parameterization of the novel of the lifetime distribution. It is supposed that the stress x_j have an effect on only the shape

parameter θ_j of the EL during a particular acceleration function. That is, $\theta_j = a + bx_j$, where the parameters, a and b are unknown based on the test method and the nature of the product.

Let the lifetime experiment be assumed under r levels of high stress x_j , j=1,2,…,r, and assume that x_u is the normal usual condition such that $x_u < x_1 < \cdots < x_r$

The pdf of EL is given by

$$f\left(t_{ij}, \beta, \lambda, \theta_j\right) = \beta\theta_j\lambda\left(1 + \lambda t_{ij}\right)^{-(\beta+1)}\left[1 - \left(1 + \lambda t_{ij}\right)^{-\beta}\right]^{\theta_j - 1},$$

$$t_{ij}, \beta, \lambda, \theta_j \geq 0,$$

(5)

The ML function is given as follows:

$$L\left(\beta, \lambda, \underline{\theta}\,; t\right) = \prod_{j=1}^{r}\prod_{i=1}^{n_j}\left[\beta\theta_j\lambda(1 + \lambda t_{ij})^{-(\beta+1)}\left[1 - (1 + \lambda t_{ij})^{-\beta}\right]^{\theta_j - 1}\right]^{\delta_{ij}}\left[1 - \left[1 - (1 + \lambda t_{ij})^{-\beta}\right]^{\theta_j}\right]^{1-\delta_{ij}},$$

(6)

where δ_{ij} is an indicator variable such that

$$\delta_{ij} = \begin{cases} 1 \ for \ t_{ij} \leq t_{cj} \\ 0 \ for \ t_{ij} > t_{cj} \end{cases},$$

Then

$$L\left(\beta, \lambda, \underline{\theta}\,; t\right) = \prod_{j=1}^{r}\prod_{i=1}^{n_j}\left[\beta\lambda\left(e^{a+bx_j}\right)(1 + \lambda t_{ij})^{-(\beta+1)}\left[1 - (1 + \lambda t_{ij})^{-\beta}\right]^{\theta_j - 1}\right]^{\delta_{ij}}$$

. (7)

It is well known that the estimators of ML, a, b, β and λ are found by maximizing the logarithm of likelihood function, symbolized by ℓ that can be printed as the following formula

$$\ell = \sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}\, ln(\beta+\lambda) + \sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}\left(a+bx_j\right) - (\beta+1)\sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}\ln\left(1+\lambda t_{ij}\right) + \sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}\left(e^{\left(a+bx_j\right)}-1\right)ln\left[1-\left(1+\lambda t_{ij}\right)^{-\beta}\right]$$

, (8)

we obtain the derivatives of ℓ respect to a, b, β and λ, as follows

$$\frac{\partial\ell}{\partial a} = \sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij} + \sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}\, e^{\left(a+bx_j\right)}ln\left[1-\left(1+\lambda t_{ij}\right)^{-\beta}\right],$$

(9)

$$\frac{\partial\ell}{\partial b} = \sum_{j=1}^{r}\sum_{i=1}^{n_j}x_j\delta_{ij} + \sum_{j=1}^{r}\sum_{i=1}^{n_j}x_j\delta_{ij}\, e^{\left(a+bx_j\right)}ln\left[1-\left(1+\lambda t_{ij}\right)^{-\beta}\right],$$

(10)

$$\frac{\partial\ell}{\partial\beta} = \frac{\sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}}{\beta} - \sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}ln\left(1+\lambda t_{ij}\right) + \sum_{j=1}^{r}\sum_{i=1}^{n_j}\frac{\left(1+\lambda t_{ij}\right)^{-\beta}\delta_{ij}\left(e^{\left(a+bx_j\right)}-1\right)ln\left(1+\lambda t_{ij}\right)}{\left[1-\left(1+\lambda t_{ij}\right)^{-\beta}\right]},$$

(11)

and

$$\frac{\partial\ell}{\partial\lambda} = \frac{\sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}}{\lambda} - \frac{(\beta+1)\sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}t_{ij}}{\left(1+\lambda t_{ij}\right)} - \frac{\beta\sum_{j=1}^{r}\sum_{i=1}^{n_j}\left(e^{\left(a+bx_j\right)}-1\right)\delta_{ij}t_{ij}\left(1+\lambda t_{ij}\right)^{-\beta-1}}{\left[1-\left(1+\lambda t_{ij}\right)^{-\beta}\right]}$$

, (12)

Therefore, the maximum likelihood

estimators (MLEs) are got by equating the first derivatives of ℓ ; to a, b, β and λ correspondingly, to zeros. As presented they are nonlinear equations and their results are numerically found by using Newton Raphson technique. They are resolved numerically to get a, b, β and λ. Depending on the invariance property of ML estimation, the shape parameter value, θ_u, in stress x_u, and the rf and hf MLEs at normal conditions when mission time t_0 could be found. The shape parameter MLEs of EL distribution θ_u, can be obtained as follows:

$$\theta_u = \exp(a + bx_u), \quad (13)$$

Besides, the rf and the hf MLEs at normal conditions when a mission time t_0 can be obtained by applying the ML invariance property in (3) and (4), respectively.

The asymptotic variance covariance matrix of the estimators a, b, β and λ, are found. The inverse asymptotic Fisher information matrix can be got from the second derivatives of the logarithm of likelihood function.

The asymptotic of Fisher information matrix $\hat{I}$ is given by:

$$\hat{I} = -\left[\frac{\partial^2 \ell}{\partial\psi_i \partial\psi_j}\right], \quad i, j = 1, 2, 3, 4, \quad (14)$$

where $\psi_1 = a,\ \psi_2 = b,\ \psi_3 = \beta \ and \ \psi_4 = \lambda,$ where the elements of the matrix (14), are

$$\frac{\partial^2 \ell}{\partial a^2} = \sum_{j=1}^{r} \sum_{i=1}^{n_j} \delta_{ij}\, e^{(a+bx_j)}\, ln\left[1 - (1 + \lambda t_{ij})^{-\beta}\right],$$

(15)

$$\frac{\partial^2 \ell}{\partial b^2} = \sum_{j=1}^{r}\sum_{i=1}^{n_j} x_j^{\,2}\delta_{ij}\, e^{(a+bx_j)} ln\left[1-(1+\lambda t_{ij})^{-\beta}\right]$$

(16)

$$\frac{\partial^2 \ell}{\partial \lambda^2} = \frac{-\sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}}{\lambda^2} + \frac{(\beta+1)\sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}\,(t_{ij})^2}{(1+\lambda t_{ij})^2} + \beta\sum_{j=1}^{r}\sum_{i=1}^{n_j}(1+\lambda t_{ij})^{-(\beta+1)}\left(e^{(a+bx_j)}-1\right)\delta_{ij}$$

(17)

$$\frac{\partial^2 \ell}{\partial \beta^2} = \frac{-\sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}}{\beta^2} - \sum_{j=1}^{r}\sum_{i=1}^{n_j} \frac{[\ln(1+\lambda t_{ij})]^2\left(e^{(a+bx_j)}-1\right)(1+\lambda t_{ij})^{-\beta}\delta_{ij}}{\left[1-(1+\lambda t_{ij})^{-\beta}\right]^2},$$

(18)

$$\frac{\partial^2 \ell}{\partial a \partial b} = \sum_{j=1}^{r}\sum_{i=1}^{n_j} x_j\delta_{ij}\, e^{(a+bx_j)} ln\left[1-(1+\lambda t_{ij})^{-\beta}\right],$$

(19)

$$\frac{\partial^2 \ell}{\partial a \partial \lambda} = \frac{\beta\sum_{j=1}^{r}\sum_{i=1}^{n_j} t_{ij}\delta_{ij}\, e^{(a+bx_j)}\left[(1+\lambda t_{ij})^{-\beta-1}\right]}{\left[1-(1+\lambda t_{ij})^{-\beta}\right]},$$

(20)

$$\frac{\partial^2 \ell}{\partial a \partial \beta} = \frac{-\sum_{j=1}^{r}\sum_{i=1}^{n_j}\delta_{ij}\, e^{(a+bx_j)}(1+\lambda t_{ij})^{-\beta}\, ln(1+\lambda t_{ij})}{\left[1-(1+\lambda t_{ij})^{-\beta}\right]},$$

(21)

$$\frac{\partial^2 \ell}{\partial b \partial \lambda} = \frac{\beta\sum_{j=1}^{r}\sum_{i=1}^{n_j} x_j t_{ij}\delta_{ij}\, e^{(a+bx_j)}\left[(1+\lambda t_{ij})^{-\beta-1}\right]}{\left[1-(1+\lambda t_{ij})^{-\beta}\right]},$$

(22)

$$\frac{\partial^2 \ell}{\partial b \partial \beta} = \frac{\sum_{j=1}^{r}\sum_{i=1}^{n_j} x_j\delta_{ij}\, e^{(a+bx_j)}(1+\lambda t_{ij})^{-\beta}\, ln(1+\lambda t_{ij})}{\left[1-(1+\lambda t_{ij})^{-\beta}\right]},$$

(23)

and

$$\frac{\partial^2 \ell}{\partial\beta\partial\lambda} = \frac{-\sum_{j=1}^{r}\sum_{i=1}^{n_j} t_{ij}\delta_{ij}}{(1+\lambda t_{ij})} + \sum_{j=1}^{r}\sum_{i=1}^{n_j} \delta_{ij}\left(e^{(a+bx_j)} - 1\right)\left[\frac{(1+\lambda t_{ij})^{-\beta} t_{ij}}{(1+\lambda t_{ij})[1-(1+\lambda t_{ij})^{-\beta}]} - \frac{(1+\lambda t_{ij})^{-\beta-1} t_{ij}\beta\ln(1+\lambda t_{ij})}{[1-(1+\lambda t_{ij})^{-\beta}]} + \frac{(1+\lambda t_{ij})^{-2\beta-1} t_{ij}\beta\ln(1+\lambda t_{ij})}{[1-(1+\lambda t_{ij})^{-\beta}]^2}\right].$$

(24)

We suppose that $A_{11} = \frac{\partial^2 \ell}{\partial a^2}$, $A_{22} = \frac{\partial^2 \ell}{\partial b^2}$, $A_{33} = \frac{\partial^2 \ell}{\partial \beta^2}$, $A_{44} = \frac{\partial^2 \ell}{\partial \lambda^2}$, $A_{12} = \frac{\partial^2 \ell}{\partial a\partial b}$, $A_{13} = \frac{\partial^2 \ell}{\partial a\partial \beta}$, $A_{14} = \frac{\partial^2 \ell}{\partial a\partial \lambda}$, $A_{23} = \frac{\partial^2 \ell}{\partial b\partial \beta}$, $A_{24} = \frac{\partial^2 \ell}{\partial b\partial \lambda}$ and $A_{34} = \frac{\partial^2 \ell}{\partial \beta\partial \lambda}$.

The asymptotic confidence limits of intervals under time censoring

On behalf of big sample size, the MLEs in suitable regularity circumstances are consistent and approximate unbiased in addition to asymptotically normally distributed. So, the two sided approximate $100(1-\alpha)\%$ confidence limits intervals for the MLE as, $\hat{w}$ of a population value w is found by $p\left(-\omega \le \frac{\hat{w}-w}{\sigma_{\hat{w}}} \le \omega\right) = (1-\varepsilon)$, where w is the $100\left(1-\frac{\varepsilon}{2}\right)$th standard normal percentile. The two sided approximate $100(1-\varepsilon)\%$ confidence limits of intervals for w, will be given as follows:

$$L_w, U_w = \hat{w} \mp \omega_{\frac{\alpha}{2}}\sigma_{\hat{w}} ,$$

(25)

where $\sigma_{\hat{w}}$ is the standard deviation, and $\hat{w}$ is $\hat{a}, \hat{b}, \hat{\beta}$ or $\hat{\lambda}$, respectively, see [14].

Optimal test plan based on time censoring

Usually the test plan gives the similar test items number to every stress. Like test plan is frequently inefficient for the mean life estimation at design stress, see [20]. In this section, we establish the statistically optimal test plan to determine that the optimal sample proportion assigned at each level of stress. Hence, to determine the optimal sample proportion r^* distributed to accelerated condition, r is selected for example, the generalized asymptotic variance (GAV) of the MLE's of the parameters is reduced. The GAV of the MLE's of the parameters for example an optimality criterion is ordinarily applied and illustrated under when the determinant reciprocal of the Fisher information matrix (see [19] and [7]).

$$GAV(\hat{a}, \hat{b}, \hat{\beta}, \hat{\lambda}) = |\tilde{I}|^{-1},$$

(26)

It is known that the GAV is minimized corresponding to the determinant is maximized of $\tilde{I}$. The technique of Newton-Raphson is employed numerically to control the rightest selection of the censoring time in every stress level that minimizes the GAV. Consequently, the equivalent optimal censoring time at each stress level is achieved by obtaining the first derivatives of $|\tilde{I}|$ to t_{c1} and t_{c2} and equating to zero

$$\frac{\partial |\tilde{I}|}{\partial t_{cj}} \; j = 1, 2.$$

(27)

where t_{cj} are the times of censoring. The determinant is achieved as follows

$|\tilde{I}| = A_{11}A_{22}A_{33}A_{44} - A_{11}A_{22}A_{34}^2 - A_{11}A_{44}A_{32}^2 + A_{11}A_{23}A_{34}A_{42} + A_{11}A_{24}A_{32}A_{43} - A_{11}A_{33}A_{42}^2 - A_{33}A_{44}A_{21}^2 + A_{12}^2A_{43}^2 + A_{12}A_{23}A_{31}A_{44} - A_{12}A_{34}A_{41}A_{32} - A_{12}A_{34}A_{42}A_{31} + A_{12}A_{24}A_{33}A_{41} + A_{13}A_{21}A_{32}A_{44} - A_{44}A_{22}A_{31}^2 + A_{13}A_{22}A_{34}A_{41} + A_{13}^2A_{24}^2 - A_{13}A_{24}A_{32}A_{41} - A_{14}A_{21}A_{32}A_{43} + A_{14}A_{21}A_{33}A_{42} + A_{14}A_{22}A_{31}A_{43} - A_{22}A_{33}A_{41}^2 - A_{14}A_{23}A_{31}A_{42} + A_{41}^2A_{23}^2$

(28)

Simulation study

A simple simulation is worked to show the theoretical outcomes of estimation and optimal test plan on the simulated data base as follows:

- Many data groups are produced from EL distribution for a arrangement of the values of the population parameters, a, b, β and λ. When size of sample 20 and 50, by applying 1000 repetitions for every sample size.

- The transformation between EL and uniform distributions is given by

$$t_{ij} = \frac{-1}{\lambda}\left(1 - (1 - u_{ij}^{\frac{1}{\theta}})^{\frac{-1}{\beta}}\right),$$

(29)

- It is supposed that simply two different stress levels, (r=2), $x_1 = 1$ and $x_2 = 1.5$, which are upper than the stress at normal condition, $x_u = 0.5$.

- The pre-determined times of censoring are $t_{c1} = 1.5$ and $t_{c2} = 3$.
- The values of population parameter for a, b, β and λ are applied at 0.5, 0.75, 2 and 1.
- A computer program R package is applied to find the derived non-linear logarithmic likelihood equation simultaneously by using the iterative technique of Newton Raphson method.
- Once the values of $\hat{a}, \hat{b}, \hat{\beta}$ or $\hat{\lambda}$ are found, these estimates are used to find, (13), and the plan stress, x_u=0.5, the stress of shape parameter θ_u ,at usual conditions is estimated as $\dot{\hat{\theta}}_u = \exp(\hat{a} + \hat{b}x_u)$. Additionally, the rf and the hf are found at several values of mission times in usual condition.
- Assessing the estimators occurrence of a, b, β and λ has been studied through some criteria's precision. So as to review the accuracy and variation of MLEs, it is appropriate to apply the relative absolute bias (RAB) $= \frac{|\text{estimate} - \text{population parameter}|}{\text{population parameter}}$, the mean square error (ER) and the relative error (RE) = $\frac{\sqrt{\text{MSE}(\text{estimate})}}{\text{population parameter}}$.
- The central asymptotic confidence limits of intervals are got for the parameters a, b, β and λ by using (25). The outcomes are shown in Tables 1-4.

Concluding remarks

- Table 1, it is obvious that the MLEs (E) improved as the sample size increase. Moreover, as appeared from the results of the numerical study that the RAB, ER and RE decrease as sample size increase. Also, the experiment at an ALT is quickly ended more than at normal circumstances.

- From Table 2, it is noted that the reliability reduces as the mission time t_0 increase. The outcomes become improved when an ALT experiment get large failures number (decrease the survival) of the device with large survival. In the other meaning, the rf increases when sample size increases. Moreover, we note that the RAB_R for the rf increases when the sample size increases. While the mission time t_0 increases, the hf decreases then increases.

- The two-sided 95% central asymptotic confidence limits of the intervals for the parameters of EL are presented in Table 3. This table shows the standard error (SE), lower limit (L), upper limit (U) and the interval length. The interval confident of the parameter gets better as the sample size increases.

- It is observed from the numerical outcomes displayed in Table 4 that the optimal test plan do not indicate the equal time of censoring at each stress level. Moreover, Table 4, contains the optimal time of censoring at each stress level for the studied several sample sizes denoted by t_{c1} and t_{c2}, and minimizes the GAV of the MLEs of the parameters as illustrated in the outcomes. The optimal GAV of the MLEs of the parameters decreases as the sample size n increases. In addition, the equivalent optimal average number of items failed at each level of stress, $\bar{r}_1^*$ and $\bar{r}_2^*$, respectively, are presented in this table.

Table 1 The E, RAB, ER and RE of the estimates at sample size 20 and 50

n	parameter	E	RAB	ER	RE
20	a	0.5628	0.1256	0.2448	0.9895
	b	0.8746	0.1661	0.2194	0.6245
	β	1.8032	0.0984	0.3682	0.3034
	λ	0.7943	0.1019	0.3817	0.2059
50	a	0.5246	0.0492	0.1682	0.8202
	b	0.7962	0.0616	0.1394	0.4978
	β	1.8983	0.0509	0.2517	0.2508
	λ	0.8859	0.0434	0.2938	0.1807

Table 2 The estimated shape parameter, rf and hr in usual condition at sample size 20 and 50

N	$\hat{\theta}_u$	t_0	$\hat{R}_u(t_0)$	RAB_R	$\hat{h}_u(t_0)$	RAB_h
20	1.0001	0.3	0.6082	0.2300	1.4518	1.0561
		0.5	0.5472	0.2761	1.2905	0.5618
		0.7	0.4506	0.2948	0.9317	0.1044
		1	0.3485	0.3009	1.0049	0.2488
50	0.9227	0.3	0.6097	0.3098	1.3932	0.4914
		0.5	0.4711	0.3768	1.2333	0.4926
		0.7	0.3760	0.4118	0.1206	0.8570
		1	0.2803	0.4377	0.9540	0.1855

Table 3 Asymptotic limits of the parameters at confidence level 95% for sample size 20 and 50

N	parameter	E	SE	U	L	length
20	a	0.5628	0.4948	1.5524	0.4268	1.1206
	b	0.8746	0.4684	1.8114	0.0622	1.7492
	β	1.8032	0.6068	3.0168	0.5896	2.4272
	λ	0.7943	0.6178	2.0287	0.4413	1.5874
50	a	0.5246	0.4101	1.3448	0.2956	1.0492
	b	0.7962	0.3734	1.5430	0.0494	1.4936
	β	1.8983	0.5017	2.9017	0.8949	2.0068
	λ	0.8859	0.5420	1.9699	0.1981	1.7717

Table 4 The results of optimal plan of the life test for sample size 20 and 50

n	r_1^*	r_2^*	t_{c1}^*	t_{c2}^*	$\vec{r}_1^*$	$\vec{r}_2^*$	GAV
20	8	8	1.1423	2.7823	7	7	0.0371
50	23	23	2.0926	5.1648	21	21	0.0069

Application I: Failure times of 84 Aircraft Windshield

In this part, we consider a real data set to compare the fits of the EL distribution. The data set consists of failure times of 84 Aircraft Windshield. The windshield on a huge aircraft is a difficult piece of equipment, contains many layers of material, counting a very sturdy outer skin with a heated layer just underneath it, all overlaid under great pressure and temperature. Failures of these items are not fundamental failures. In its place, they usually include damage or delamination of the non- essential outer layer or failure of the heating scheme. These failures do not result in harm to the aircraft but perform outcome in replacement of the windshield. We study the data on failure times for a exact model windshield shown in Table 16.11 of [13]. Many recent studies, [16] and [9] have studied these data.

The failure times of 84 Aircraft Windshield is

0.040, 1.866, 2.385, 3.443, 0.301, 1.876, 2.481, 3.467, 0.309, 1.899, 2.610, 3.478, 0.557, 1.911, 2.625, 3.578, 0.943, 1.912, 2.632, 3.595, 1.070, 1.914, 2.646, 3.699, 1.124, 1.981, 2.661, 3.779,1.248, 2.010, 2.688, 3.924, 1.281, 2.038, 2.82,3, 4.035, 1.281, 2.085, 2.890, 4.121, 1.303, 2.089, 2.902, 4.167, 1.432, 2.097, 2.934, 4.240, 1.480, 2.135, 2.962, 4.255, 1.505, 2.154, 2.964, 4.278, 1.506, 2.190, 3.000, 4.305, 1.568, 2.194, 3.103, 4.376, 1.615, 2.223, 3.114, 4.449, 1.619, 2.224, 3.117, 4.485, 1.652, 2.229, 3.166, 4.570, 1.652, 2.300, 3.344, 4.602, 1.757, 2.324, 3.376, 4.663.

The population parameter values is applied for this data, a=0.5, b=0.5, β=2 and λ=0.5. The connection between the shape parameter and stress is examined during testing the coefficient significance b. Hypothesis test is got at $\alpha = 0.05$ and when the degree of freedom equal one, supposing that the null hypothesis is b=0. It is not accepted and the correlation between the stress level and the shape parameter exists. The optimal test plan is at GAV=0.0849.

Application II: Service times of 63 Aircraft Windshield

This application illustrate the service times. These real data consist of 63 aircraft windshield service times; see [13] and [18] as follows:

0.046, 1.436, 2.592, 0.140, 1.492, 2.600, 0.150, 1.580, 2.670, 0.248, 1.719, 2.717, 0.280, 1.794, 2.819, 0.313, 1.915, 2.820, 0.389, 1.920, 2.878, 0.487, 1.963, 2.950, 0.622, 1.978, 3.003, 0.900, 2.053, 3.102, 0.952, 2.065, 3.304, 0.996, 2.117, 3.483, 1.003, 2.137, 3.500, 1.010, 2.141, 3.622, 1.085, 2.163, 3.665, 1.092, 2.183, 3.695, 1.152, 2.240, 4.015, 1.183, 2.341, 4.628, 1.244, 2.435, 4.806, 1.249, 2.464, 4.881, 1.262, 2.543, 5.140

The values of population parameter are a=1.5, b=2, β=3 and λ=1.5, we study the relationship between the stress and the shape parameter through the coefficient of significance, b. we test this hypothesis test at a=0.05 and degree of freedom equal one, we assume that the null hypothesis is b= zero. The null hypothesis is not reject so there is relation between the stress level and the shape parameter. The optimal test plan of these data is at GAV=0.00972.

Data Availability

The data used to support the findings of this study are included within the article

Conflict of Interests

The authors declare that there is no conflicts of interest regarding the publication of this paper**.**

References

[1] Abdel-Hamid, A. & AL-Hussaini, E. 2009. Estimation in step stress accelerated life tests for the exponentiated exponential distribution with Type I censoring. *computational statistics and data analysis 53*: 1328-1338.

[2] AL-Hussaini, E. & Abdel-Hamid, A. 2004. Bayesian estimation of the parameters, reliability and hazard rate functions of mixtures under accelerated life tests. *communication in statistics-simulation and computation 33 i4*: 963-982.

[3] AL-Hussaini, E. & Abdel-Hamid, A. 2006. Accelerated life tests under finite mixture models. *journal of statistical computional simulation 76 i8*: 673-690.

[4] AL-Dayian, G., EL Helbawy, A. & Rezk, H. 2014. Statistical inference for a simple constant stress model based on censored sampling data from the Kumaraswamy Weibull distribution, *international journal of statistics and probability, 3,* 80-95.

[5] Attia, A. F., Aly, H. M. & Bleed, S. O. 2011. Estimating and

planning accelerated life test using constant stress for generalized logistic distribution under Type-I censoring. *applied mathematics*. 1-15.

[6] Bagdonavicius, V. & Nikulin, M. 2002. *Accelerated life models: modeling and statistical analysis*. Chapman & Hall-CRC press, Boca, Raton, Florida.

[7] Bai, D. S. & Chung,S. W. 1992. Optimal design of partially accelerated life tests for the exponential distribution under type-I censoring. *IEEE Transactions on Reliability 41*: 400-406.

[8] Gouno, E., Sen, A. & Balakrishnan, N. 2004. Optimum simple ramp-tests for the Weibull distribution and type-I censoring. *IEEE transaction on reliability* 41: 407-413.

[9] El-Bassiouny, A. H., Abdo, N. F. & Shahen, H. S. 2015. Exponential Lomax distribution. *international journal of computer applications* 121: 24-30.

[10] Escobar, L. A. & Meeker, W. Q. 1995. Planning accelerated life tests with two more factors. *technometrics 37*: 411-427.

[11] Lomax, K. S. 1954. Business failures; another example of the analysis of failure data. *journal of the american statistical association 49:* 847-852.

[12] Meeker, Q. W. & Escobar, L. A. 1998. Accelerated degradation tests: modeling and analysis *technometrics 40*: 89-99.

[13] Murthy, D., Xie, M. & Jiang, R. 2004. *Weibull models*. John Wiley & Sonc, Inc.

[14] Nelson, W. B. 1982. *Applied life data analysis*. New York: John Wiley.

[15] Nelson, W. B.1990. *Accelerated testing statistical models, test plans and data analysis*. John Wiley & Sonc, Inc.

[16] Ramos, M. W. A., Marinho, P. R. D., da Silva, R. V. &

Cordeiro, G. M. 2013.The exponentiated Lomax Poisson distribution with an application to lifetime data. *advances and applications in statistics 34*: 107-135.

[17] Salem, H. 2014. The exponentiated Lomax distribution: different estimation methods. *american journal of applied mathematics and statistics 2*: 364-368.

[18] Tahir, M. H. , Gauss M. Cordeiro , Mansoor, M. and Zubair, M. (2015). The Weibull-Lomax distribution: properties and applications, *Hacettepe Journal of Mathematics and Statistics*, 44: 455 - 474

[19] Wu, S., Lin, Y. & Chen, S. 2008. Optimal step stress test under Type I progressive group censoring with random removals. *journal of statistical planning and inference 138*: 817-826.

[20] Yang, G. B. 1994. Optimum constant-stress accelerated life-test plans. *IEEE transaction on reliability 43*: 575-581.