# Computer Simulation System for Queueing Coronavirus 2019 (COVID-19) Patients in a Governmental Emergency Unit Using Queueing Theory

**Howaida Elsayed[1] Mohamed Alghobiri[2] Waleed Elsayed[3]**

**1- Statistics Department, Business College, King Khalid University, ABHA, Saudi Arabia**

**2- Computer Science Department, Business College, King Khalid University, ABHA, Saudi Arabia**

**3- Administration Department, Commerce College, Benha University, Benha, Egypt**

**Abstract**

This paper addresses the problem of potential overcrowding by patients in the emergency unit of the Asir governmental hospital located in Abha city, Saudi Arabia. This issue affects the patient waiting time, the level of service provided, the quality of service, and the level of pressure on the medical staff; thus, waiting lines are formed, for various reasons. The main objective of this paper is to reduce the waiting time of patients and to increase the efficiency of the emergency unit. This paper presents an attempt to apply queuing theory to an emergency unit; the calculations made in this paper are based upon actual observed data collected from the emergency unit in the Asir governmental hospital to identify the appropriate distributions for access and service times. The research serves as the basis for a computer simulation model to accurately represent the current reality of the system, to assist decision-makers in overcoming the wait time problem to reduce patient waiting time to zero minutes. Actual data and simulation results revealed the waiting time and number of patients in the queue decreased upon the application of queueing theory.

**Keywords and Phrases:** Queueing Theory, Arrival Rate, Service Time Rate, Exponential Distribution, Poisson Distribution, Quality of Health Service, Simulation.

**MSC2020 Mathematics Subject Classification:** 60E05, 60K25, 62G10, 68M20, 90B22.

## 1. Introduction

The continuing global prevalence of Coronavirus Disease 2019 (COVID-19) is placing significant pressure on healthcare systems. The purpose of this study is to reduce the time patients have to wait to be treated and also to increase the efficiency of the hospital emergency unit. The oldest and most widely used quantitative analysis technique for addressing waiting time phenomenon from a mathematical perspective is queueing theory [3,4]. Queueing theory was developed by the Danish engineer A.K. Erlang [1], and is a useful tool for making decisions in governmental hospitals to determine specific needs and resources, such as the number of doctors per unit, number of medical devices, the number of waiting rooms, and so forth. Hospital decision-makers seek to ensure the wait times before receiving service are as short as possible, to ensure patients are satisfied with the service and that they do not leave the hospital untreated [5,10].

The quality of health services is a key component of the right to health, which means the provision of health services that are safe, easily accessible, streamlined for providers, considered satisfactory by those who benefit, and generate a positive view of healthcare in the community [7,14]. The World Health Organization (WHO) defines quality health services as those consistent with the correct criteria and trends, delivered in a safe way that is accepted by the community and at an affordable cost, such that they lead to a significant impact on the proportion of comorbidities and mortality [18]. To achieve universal health coverage in the emergency unit of the Asir governmental hospital located in Abha city, Saudi Arabia, it is essential to deliver health services that meet these quality criteria.

Waiting lines are a part of life; customers are required to wait in line to access multiple services, e.g. waiting to eat in restaurants, queueing at check-out counters in grocery stores, cars waiting at traffic lights, and patients waiting to be treated in hospital. Queuing theory determines performance measures of wait time, such as such as average queue length, average time waiting in a queue, and average facility utilization. Queuing models are a constituent of queuing theory; a model will be constructed so that queue lengths and waiting time can be estimated. Queuing theory is used in multiple fields, such as medicine, trade, and transportation, to improve the design of processing systems [9].

In this study, an attempt is made to apply queuing theory to solving the problem of overcrowding of potential patients in the emergency unit of the Asir governmental hospital located in Abha city, Saudi Arabia, which affects the patients' wait time, the level of service provided, the quality of service, and the level of pressure on the medical staff. Solving this issue will reduce the wait time for patients and increase the efficiency of the emergency unit. In this study, real hospital data is used; hourly data was collected from the emergency unit, where patients' arrival time and service time were recorded for 7 working days (24 hours a day) from 5 July 2020 to 11 July 2020, and performance measures were determined. The research aimed to build a computer simulation model, creating an honest representation of the reality of the system, to help decision-makers eliminate the waiting period. The model was built using the QSB program, and a queuing model was employed for this study.

The paper is organized as follows: Section 2 will present the research problem and objectives, and Section 3 will discuss the research methodology and the statistical analysis of real data, and a simulation study using the WINQSB program is presented in Section 4. Discussion is presented in Section 5, and Section 6 presents the conclusion of the paper.

## 2. Research Problem and Objectives

### *2.1 Research Problem*

This research aims to solve the problem of queues experienced by potential patients in the emergency unit of Asir governmental hospital located in Abha city, Saudi Arabia, in order to reduce patient wait times and the number of patients waiting, and thus improve the efficiency of the service provided. To achieve this, the researcher will seek to answer the following main question:

Is it possible to build a simulation model for patient queues in emergency units to minimize waiting time?

The main research question can be divided into the following sub-questions:

1- What is the probability distribution of the arrival of potential Coronavirus (COVID-19) patients to emergency units?
2- What is the probability distribution of the service time of potential COVID-19 patients to emergency units?
3- Is it possible to build a simulation model to serve potential COVID-19 patients so that wait times can be reduced to zero minutes?

### *2.2 Research Objectives*

The main objectives of this paper concerning the emergency unit are the following:

1- To determine the rate of arrival of potential COVID-19 patients and their statistical distribution.
2- To determine the rate of service of potential COVID-19 patients and their statistical distribution.
3- To build a computer simulation model, an honest representation of the reality of the system, to help decision-makers overcome the waiting phenomenon so that the wait time becomes zero minutes.
4- To identify performance measures and choose an optimal queuing model so that the service potential COVID-19

patients receive is satisfactory to patients in the emergency unit and to the hospital administration.

## 3. Research Methodology

The methodology used in the research was based on a descriptive approach to the theoretical framework regarding queuing theory, and a quantitative analytical approach to study factors related to reducing the wait time of patients and increasing the efficiency of the emergency unit using queuing models. This was achieved by collecting and studying statistical data on the arrival of potential COVID-19 patients at the emergency unit from 5 July 2020 to 11 July 2020, where hourly data was collected at the emergency unit of Asir governmental hospital located in Abha city, Saudi Arabia. Specifically, patients' arrival time and service time were recorded for 7 working days (24 hours a day).

### *3.1 Theoretical Framework*

#### *3.1.1 Distributions*

In this study, models that follow the Poisson distribution for patients' accessing the process of admission and the exponential distribution of service times are used. The following presents a brief overview of each of the above distributions [6,17].

#### 1-Poisson Distribution

The pattern of patient access to the system can be in the form of equal time periods or unequal time periods, in other words, random arrival(X). Therefore, probability distributions were used to describe the access rate; the most widely used probability distribution is the Poisson distribution. A discrete random variable (i.e. the number of arriving patients) is said to have a Poisson distribution within certain parameters; the probability density function of X is given as:

$$P_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0,1,2,\ldots \quad ,\lambda > 0 \qquad (1)$$

where

x is the number of occurrences (i.e., the number of patients arriving in the emergency unit

$\lambda$: the expected value of X (arrival rate )

The mean or expected value and variance of patient access are symbolized by E(X) and Var (X), respectively.

E(X)=Var (X)= $\lambda$

e indicates Euler's number (e = 2.71828...)

The arrivals of patients occur independently.

**2- Exponential Distribution**

The pattern of patients leaving the system, which represents the service time – i.e., the time period between two consecutive services –may be fixed or random. Most queuing models require that the service rate is randomly distributed probability density according to the exponential distribution. The of an exponential distribution has the following formula: function

$$f_T(t;\lambda) = \mu e^{-\mu t}, \qquad t \geq 0$$

of an exponentially distributed random expected valueThe variable *t* with the parameter λ is given as:

$$E(T) = \mu = \frac{1}{\lambda}, \qquad \lambda > 0$$

is the parameter of the distribution, often called $\lambda$where the rate parameter.

### 3.1.2 Queueing System Models

D. G. Kendall [12] suggested describing queuing models using three notations (A/S/C), as appropriate, to summarize the characteristics of queuing systems [16]. This has since been extended to include three further notations (D/N/K). Thus, currently, six notations are used by researchers to describe queuing systems, with the following model format:

**(A/S/C) :(D/N/K)**

Where
**A**: Arrivals distribution
**S**: Service time distribution
**C**: Number of service channels (or servers) (C=1, 2, …, ∞)
D: The queuing discipline
**N**: The capacity of the system (in-queue plus in-service); that is, the maximum number of patients allowed in the queue (the capacity is assumed to be unlimited)
**K**: The size of the calling source (the size of the population from which the patients come, assumed to be unlimited)

Kendall's notation is the standard system used to describe and classify a queuing node. The standard notations for representing the arrivals and service time distributions (i.e., **A** and **S** symbols) are:
**M** = Poisson distribution of inter-arrivals or exponentially distributed service time
**D** = Deterministic arrivals and service times
**U** = Uniform distribution of inter-arrivals or service time
**Ek** = Erlang distribution of inter-arrivals or service time
**GI** = General distribution of inter-arrivals
**G** = General distribution of service time

### 3.1.3 Service Channels

Queuing systems are either single server (single channel) or multi server (multichannel) and the client can receive the service in one or several phases. The system serving capacity is a function of the number of service facilities and server efficiency [11, 9].

Service systems are usually classified according to the number of channels that provide the service (the number of service stations) and the number of phases (number of stops), and therefore can be distinguished according to four forms of waiting line (queue).

## 1. Single-server and Single-phase Queuing System

In this type of system, it is assumed that a server or channel can serve one patient at a time and provide the service in one phase. The patients enter the system (i.e., the emergency unit) and request the service. If the system is free, the patient service time begins immediately; when the system is not free, the patient stands in a queue and waits for the server to be free to provide the service, which is delivered in one phase, as shown in (Fig.1).

## 2. Single-server and Multi-phase Queuing System

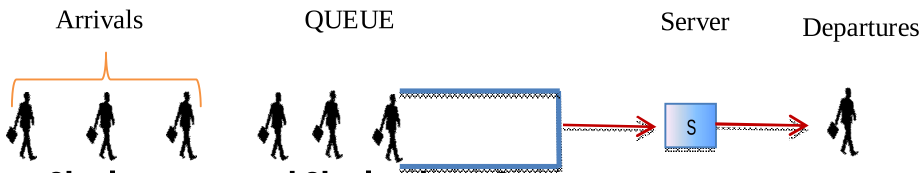
In this case, it is assumed that a server can serve one patient at a time and the patient goes through several phases to receive the full service, as shown in (Fig.2).

## 3. Multi-server and Single-phase Queuing System

In this case, each channel performs the same service as the other channels and with the same efficiency, meaning more than one customer can be serviced at the same time, as shown in (Fig.3).

## 4. Multi-server and Multi-phase Queuing System

Each channel performs the same service as the other channels, and the customer goes through several stages to receive the service, as shown in (Fig.4).




. Single-server and Single-phase System Fig.1

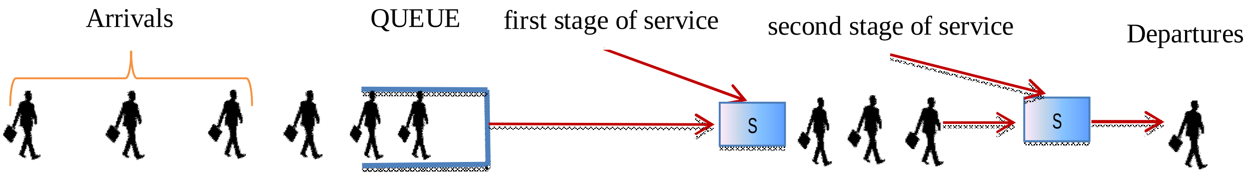



Fig.2. Single-server and Multi-phase System

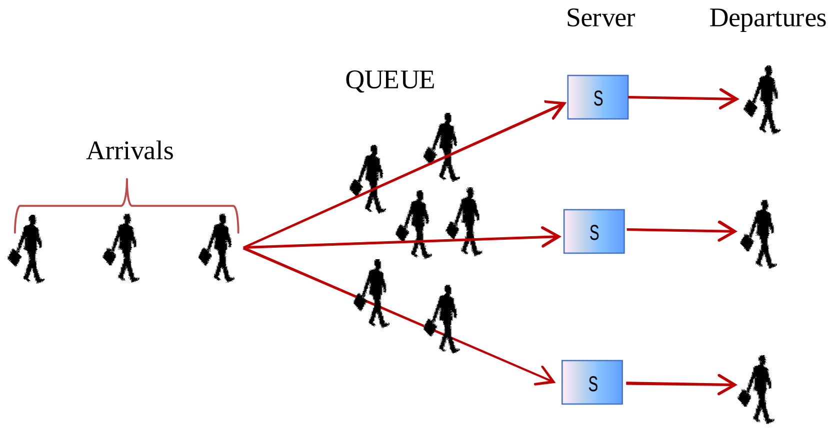



Fig.3. Multi-server and Single-phase System

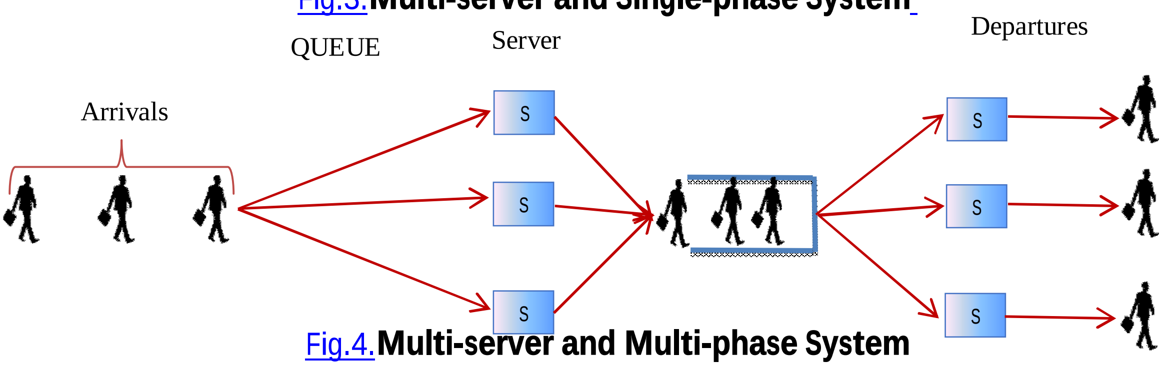



Fig.4. Multi-server and Multi-phase System

### 3.1.4 Queue Discipline

Queue discipline (i.e., D symbol) is
The server accepts patients for service according to the following rules:
**FCFS:** First come, first served
**LCFS:** Last come, first served
**RS:** Random selection for service
**PS:** Priority service (i.e., servicing those patients that have priority over others according to certain criteria)

### 3.1.5 Performance Measures of the Queuing System

Performance measures are a way of analyzing the efficiency of the queuing system under consideration and are used in the decision-making process. The most commonly used measures of performance are[13,15]:
$\lambda$: Average arrival rate of patients
$\mu$: Average service rate of patients
$\rho$: Traffic intensity (i.e., the probability of queuing on arrival)

Traffic intensity is a measure of the average occupancy of the facility during a specified period of time; it is obtained from dividing the average arrival rate $\lambda$ by the average service rate $\mu$. The formula is:

$$\rho = \frac{\lambda}{\mu}$$

If the rate of patients arriving to the system is greater than the rate of service, i.e. $\lambda > \mu$ then $\rho > 1$, which means the system capacity is less than the arriving patients; thus, the queue length is increased.

$\rho_0$: The probability of not queuing on arrival (i.e., the probability of there being zero patients in the system), is defined as:

$$\rho_0 = 1 - \rho$$

$L_q$: Expected number of patients in the queue, defined as:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\rho^2}{(1-\rho)}$$

$L_s$: Expected number of patients in the system who are in the queue plus those being served, defined as:

$$L_s = \frac{\lambda}{(\mu - \lambda)} = \frac{\rho}{(1-\rho)}$$

$W_q$: Expected waiting time of the patients in the queue before being provided with the services which they are waiting for, defined as:

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{\rho}{\mu(1-\rho)}$$

$W_s$: Expected time that the patients spend in the system, which includes the wait time and service time. The formula for this is:

$$W_s = \frac{L_s}{\lambda} = \frac{1}{(\mu - \lambda)} = \frac{1}{\mu(1-\rho)}$$

**3. 2 Applied Study on Real Data**

The statistical study was conducted as follows:

***3.2.1 Data Collection***

The arrival records and service times of patients were collected from the emergency unit of the Asir governmental hospital located in Abha city, Saudi Arabia. The total period of the observation was one week from Sunday 5 July 2020 to Saturday 11 July 2020 throughout the whole day. The data consisted of 24 observations per day (one observation per hour), with a total number of observations for the week estimated to be 186.

### *3.2.2 Data Analysis*

Before determining the appropriate type of queuing model, one must first establish which of the probability distributions each of the arrival times and service times follows. Then, performance measures of the queuing system can be established.

#### ***3.2.2.1 Arrival distribution of patients at the emergency unit of the Asir governmental hospital***

The sample that was used to determine the type of probability distribution the arrival rate followed is described in the Table 1:

**Table 1. Frequency distribution of patient arrival during the observation period at the emergency unit**

| **No. of class intervals k** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | **Total** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frequency** | 11 | 16 | 30 | 31 | 27 | 21 | 14 | 7 | 7 | 2 | 2 | **168** |

**Source: Prepared by the researcher**

In order to determine the rate of arrival of potential COVID-19 patients and their statistical distribution, several steps were followed:

**Step 1: Determining statistical hypotheses**

The following hypotheses regarding the distribution of patient arrival were tested:

**Null Hypothesis:** The arrival of patients follows a Poisson probability distribution

**Alternative Hypothesis:** The arrival of patients does not follow a Poisson probability distribution

**Step 2: Computing the arrival rate of patients**

Using the above table, we computed the arrival rate of the patients $\lambda$ as follows:

$$\lambda = \frac{\sum_{i=1}^{k} x_i O_i}{n} = \frac{609}{168} = 3.63 \ patient \ per \ hour$$

**Step 3: Computing the probability density function of patient access**

There is a theoretical hypothesized probability associated with i^th^ class interval, i.e. Pi, which is calculated using Eq. (1) and the POISSON.DIST (x, mean, cumulative) function in Excel.

**Step 4: Calculate a Chi-square test statistic** $\chi^2_{calculated}$

We then applied the Chi-square goodness of fit to test the hypothesis that the underlying distribution is Poisson, with the level of significance set as =0.05. The test procedure began by arranging n =168 observations into a set of k class (k=11) intervals. We then computed the Chi-square goodness of fit test statistics using the following formula[2]:

$$\chi^2_{calculated} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

Where

$O_i$ : Observed frequency of arrival patients in the $i^{th}$ class interval

$E_i$: Expected frequency of arrival patients in the $i^{th}$ class interval, which is computed as:

$$E_i = nP_i$$

The following table shows the results of the Chi-square goodness of fit test.

Table 2:*Chi-square goodness of fit test*

| No. of Intervals $x_i$ | Observed Frequency $O_i$ | $X_i*O_i$ | Patient Arrival Rate | Probability Poisson Distribution $P_i$ (x, mean, cumulative) | *Expected Frequency $E_i=n*P_i$ | $x_i$ | $O_i$ | $E_i$ | Chi-square Calculated Value |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 11 | 0 | 3.63 | 0.026649097 | 4.47704835 | 1 | 27 | 20.706349 | 1.9129422 |
| 1 | 16 | 16 | 3.63 | 0.096602978 | 16.2293003 | | | | |
| 2 | 30 | 60 | 3.63 | 0.175092897 | 29.4156068 | 2 | 30 | 29.415607 | 0.01161 |
| 3 | 31 | 93 | 3.63 | 0.211570584 | 35.5438582 | 3 | 31 | 35.543858 | 0.580878 |
| 4 | 27 | 108 | 3.63 | 0.191735842 | 32.2116215 | 4 | 27 | 32.211621 | 0.8432049 |
| 5 | 21 | 105 | 3.63 | 0.139008485 | 23.3534256 | 5 | 21 | 23.353426 | 0.2371649 |
| 6 | 14 | 84 | 3.63 | 0.083984293 | 14.1093613 | 6 | 14 | 14.109361 | 0.0008477 |
| 7 | 7 | 49 | 3.63 | 0.043491866 | 7.30663352 | $\geq 7$ | 18 | 12.434378 | 3.5866477 |
| 8 | 7 | 56 | 3.63 | 0.019707252 | 3.31081831 | | | | |
| 9 | 2 | 18 | 3.63 | 0.007937643 | 1.33352404 | | | | |
| 10 | 2 | 20 | 3.63 | 0.002877396 | 0.48340247 | | | | |
| **Total** | **168** | **609** | | **0.998658334** | 167.774600 | | **168** | **167.7746** | **7.173295** |

**Source: Prepared by the researcher**

* The expected frequencies should be greater than 5 for , $E_i$ and $E_i \geq 5$ calculation (assumption of Chi-Square test). To get corresponding $O_i$ are combined where they are less than 5.

**Step 5: Construct acceptance/rejection regions**

**Critical Value :**

$$\chi^2_{Critical} = \chi^2_{(\alpha,v)} = \chi^2_{(0.05,5)} = 11.7 \ \ (from\ the\ \ chi - square\ table)$$

$k$ = class intervals after being combined

$s$ = 1 as it was necessary to estimate one parameter from the data

=7-1-1=5$v =$ $Degree\ of\ freedom\ is\ (k - s - 1)$

**Calculated value:**

$\chi^2_{calculted} = 7.17$

Since $\chi^2_{calculted} = 7.17 < \chi^2_{Critical}$=11.7, $H_0$ is accepted. Hence, the arrival of patients can be said to follow a Poisson probability distribution.

**Step 6: Decision processes**

Based on the Chi-square goodness of fit test, there is no evidence requiring rejection of the null hypothesis, which states that the distribution of patient arrivals follows a Poisson distribution with an arrival rate equal to 3.63 patients per hour.

### *3.2.2.2 Service time distribution of patients at the emergency unit of the Asir governmental hospital*

Service time is a random, not fixed, variable, and differs from patient to patient. In order to determine the service rates of potential COVID-19 patients and their statistical distribution, we followed the same steps as were done previously to identify the probability distribution of patient arrivals, as follows:

**Step 1: Determining statistical hypotheses**

The following hypotheses were tested with regard to the distribution of service times:

**Null Hypothesis:** The service times follow an exponential probability distribution

**Alternative Hypothesis:** The service times do not follow an exponential probability distribution

**Step 2: Compute the average service time of patients**

1- We applied Sturge’s rule to determine the number of intervals in the data, as follows:

$$k = 1 + 3.3 \, log \, log \, n$$

$$k = 1 + 3.3 \, log \, log \, (168) = 8.34 \cong 8 \, intervals$$

where:

$k$: *number of intervals*

n: observation number

2- We then calculated the width of the interval using the following formula:

$$width\ of\ the\ interval = \frac{Max.observed\ service\ time - Min.bserved\ service\ time}{k}$$

$$width\ of\ the\ interval = \frac{8.2044 - 0.0084}{8} = 1.0245$$

**Table 3.** *Chi-square goodness of fit test*

| Service Time Intervals | Observed Frequency $O_i$ | Interval Centre $x_i$ | $X_i*O_i$ | Probability Exponential Distribution $P_i$ | Expected Frequency $E_i=n*P_i$ | $O_i$ | $E_i$ | Chi-Square Calculated Value |
|---|---|---|---|---|---|---|---|---|
| 0.0084 -1.0329 | 59 | 0.52065 | 30.71835 | 0.334198 | 56.14532 | 59 | 56.14532 | 0.1451 |
| 1.0329- 2.0574 | 34 | 1.54515 | 52.5351 | 0.218508 | 36.7094 | 34 | 36.7094 | 0.2000 |
| 2.0574-3.0819 | 25 | 2.56965 | 64.24125 | 0.142867 | 24.00165 | 25 | 24.00165 | 0.0415 |
| 3.0819-4.1064 | 17 | 3.59415 | 61.10055 | 0.09341 | 15.69296 | 17 | 15.69296 | 0.1089 |
| 4.1064-5.1309 | 11 | 4.61865 | 50.80515 | 0.061074 | 10.2605 | 11 | 10.2605 | 0.0533 |
| 5.1309-6.1554 | 7 | 5.64315 | 39.50205 | 0.039932 | 6.708606 | 7 | 6.708606 | 0.0127 |
| 6.1554-7.1799 | 9 | 6.66765 | 60.00885 | 0.026109 | 4.386277 | 15 | 7.254149 | 8.2709 |
| 7.1799-8.2044 | 6 | 7.69215 | 46.1529 | 0.017071 | 2.867872 | | | |
| **Total** | **168** | | **405.0642** | **0.93317** | | | | **8.8323** |

**Source: Prepared by the researcher**

(2-1) * The expected frequencies should be greater than 5 for calculation (assumption of Chi-square test). To get $E_i \geq 5$, $E_i$ and corresponding $O_i$ were combined where they had a value less than 5.

From the above table, the service rate and exponential parameter $\mu$ be calculated as follows:

$$\lambda = \frac{\sum_{i=1}^{k} x_i O_i}{n} = \frac{405.0642}{168} = 2.39 \ service\ per\ hour$$

$$\mu = \frac{1}{\lambda} = \frac{1}{2.39} = 0.418 \ service\ per\ hour$$

**Step 3: Compute the probability density function of patient service time**

The theoretical hypothesized probability associated with i[th] class interval, i.e. Pi, which was calculated using Eq. (2) as shown in Table 2.

**Step 4: Calculate the Chi-square test statistic $\chi^2_{calculated}$**

We then applied the Chi-square goodness of fit statistic to test the hypothesis that the underlying distribution is exponential, with the level of significance set at α=0.05. Using the data in Table 2, the chi-square test statistic was calculated as follows:

$$\chi^2_{calculated} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 8.8323$$

**Step 5: Construct acceptance/rejection regions**

**Critical value:**

$\chi^2_{Critical} = \chi^2_{(\alpha,v)} = \chi^2_{(0.05,5)} = 11.7 \ (from\ the\ chi-square\ table)$

$k$ = class intervals after being combined

$s$ = 1 as it was necessary to estimate one parameter from the data

$v = \ Degree\ of\ freedom\ is\ (k - s - 1)$=7-1-1=5

**Calculated value:** $\chi^2_{calculted} = 7.17$
Since $\chi^2_{calculted} = 8.323 < \chi^2_{Critical}$=11.7, $H_0$ is accepted.
Hence, it can be said that the service times follow an exponential probability distribution.

**Step 6: Decision processes**

Based on the Chi-square goodness of fit test, there is no evidence to reject the null hypothesis which states that the distribution of service times follows an exponential distribution, with a $\mu = 0.418$ service rate per hour.

In analyzing the results presented in Tables 1 and 2, we observed that:
1- The distribution of patients' arrival in the emergency unit of the Asir governmental hospital followed a Poisson distribution pattern with an arrival rate of 3.63 patients per hour.
2- The distribution of service times for patients in the emergency unit of Asir governmental hospital followed an exponential distribution pattern with a $\mu = 0.418$ service rate per hour.
3- Services were provided to patients on a first-come, first-served basis.
Thus, the appropriate mathematical model with which to study the queuing system is:

$$(M\backslash M\backslash\ 1)(FCFS\backslash\infty\backslash\infty)$$

## 4. Simulation Study

This section focuses on building a computer simulation model, an honest representation of the reality of the system to help decision-makers overcome the waiting problem so that the wait time becomes zero minutes. This goal will be achieved out through studying several simulations. Various service channels were considered, using the WINQSB program, and $(M/\ M/1)(FIFO/\infty/\infty)$ was deemed the most appropriate mathematical model to be applied to study the system.

After determining the hourly patient arrival rate (Lambda) to be =3.63 patient/hour, the service rate per server(mu) per hour =0.418 service/hour, and the number of service channels assumed in the study(C=1), we then analyzed the efficiency of the queuing system and extracted the performance measures using the WINQSB program, as follows **Table 4**:

**Table 4**: *Performance measures of the queuing system using the WINQSB program*

| | Performance Measure | Result |
|---|---|---|
| 1 | **System: M/M/1** | **From Simulation** |
| 2 | Patient arrival rate (Lambda) per hour= | 3.6300 |
| 3 | Service rate per server(mu) per hour= | 0.4160 |
| 4 | Overall system effective arrival rate per hour= | 3.5956 |
| 5 | Overall system effective service rate per hour= | 0.4138 |
| 6 | Overall system utilization= | 99.9820% |
| 7 | Average number of patients in the system(L)= | 1583.2150 |
| 8 | Average number of patients in the queue (Lq)= | 1582.2150 |
| 9 | Average number of patients in the queue for a busy system(Lb)= | 1582.4990 |
| 10 | Average time patients spends in the system(W)= | 451.2919 hours |
| 11 | Average time patients spends in the queue (Wq)= | 448.8826 hours |
| 12 | Average time patients spends in the queue for a busy system(Wb)= | 448.9634 hours |
| 13 | The probability that all servers are idle($P_0$)= | 0.0180% |
| 14 | The probability an arriving patient waits (Pw) or system in busy (Pb)= | 99.9820% |
| 15 | Average number of patiants being balked per hour= | 0 |
| 16 | Total cost of busy server per hour = | 0$ |
| 17 | Total cost of idle server per hour = | 0$ |
| 18 | Total cost of patient waiting per hour = | 0$ |
| 19 | Total cost of patient being served per hour = | 0$ |
| 20 | Total cost of patient being balked per hour = | 0$ |
| 21 | Total queue space cost per hour= | 0$ |
| 22 | Total system cost per hour= | 0$ |
| 23 | Simulation time in hour= | 1000.0000 |
| 24 | Starting time in hour= | 0 |
| 25 | Number of observations collected= | 414 |
| 26 | Maximun number of patients in the queue= | 3182 |
| 27 | Total simulation CPU time in second= | 0.2480 |

**Source: Win QSB Program Outputs**

When analyzing the results shown in Table 4, we observed that the traffic intensity was 99.98%, which indicates overcrowding of potential patients in the emergency unit of Asir governmental hospital. This represents an inadequate service system for the emergency unit. Hence, we recommend adding additional service channels. The expected number of patients in the queue is 1,582 patients, a large number. The expected number of patients in the system, being those in the queue plus those being served, is 1,583 patients, again a large number. The expected wait time for the patients in the queue before they are provided with the service, they are waiting for is to 448 hours, which indicates that the emergency unit is suffering from a severe waiting phenomenon. The decision-maker should study this phenomenon and seek to reduce the wait time for patients and increase the efficiency of the emergency unit to the greatest extent possible, as the current expected wait time for the patients in the system is equal to 451 hours.

The simulation results showed that the emergency unit suffers from a waiting phenomenon. Decision-makers must overcome this problem by adding service channels to minimize the wait time of patients and to increase the efficiency of the emergency unit. Table 5 shows four scenarios, where 5, 9, and 12 additional service channels are added to the system (C=1, C=6, C=10, C=13). The fourth scenario is clearly preferable to the first, second, and third scenarios because when 12 service channels are added, the traffic intensity decreases from 99.98% to 67.12%, which indicates a decrease in overcrowding of potential patients in the emergency unit, the probability of the system being idle is 32.88%, and the wait time reduced to 0.07 hours. From the above results, the recommended solution is fourth scenario as all performance measures supporting its application **Table 5**.

**Table 5**: *Performance measures of the queuing system*

| Scenario | No. of servers C | rate $\lambda$ | Service rate $\mu$ | Traffic intensity $\rho$ | [1]Probability of system idleness $\rho_0$ | [2]E(X) in the system $L_s$ | [3]E(X) in the queue $L_q$ | [4]E(T) ) in the system $W_s$ | [5]E(T) ) in the queue $W_q$ |
|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 3.63 | 0.416 | 99.98% | 0.02% | 1,583 | 1,582 | 451 | 449 |
| **2** | 6 | 3.63 | 0.416 | 99.68% | 0.32% | 567 | 562 | 162 | 160 |
| **3** | 10 | 3.63 | 0.416 | 87.26% | 12.74% | 13 | 4 | 3.5 | 1.1 |
| **4** | 13 | 3.63 | 0.416 | 67.12% | 32.88% | 9 | 0.27 | 2.5 | 0.07 |

**Source**: Win QSB Program Outputs

[1] The probability of the system being idle
[2] Expected number of patients in the system
[3] Expected number of patients in the queue
[4] Expected wait time of the patients in the system
[5] Expected wait time of the patients in the queue

## 5. Discussion

The continuing global prevalence of Coronavirus Disease 2019 (COVID-19) is placing significant pressure on healthcare systems. This paper addresses the problem of potential overcrowding by patients in the emergency unit of the Asir governmental hospital located in Abha city, Saudi Arabia. This issue affects the patient waiting time, the level of service provided, the quality of service, and the level of pressure on the medical staff; thus, waiting lines are formed, for various reasons. Solving this issue will reduce the wait time for patients and increase the efficiency of the emergency unit. In this study, real hospital data is used; hourly data was collected from the emergency unit, where patients' arrival time and service time were recorded for 7 working days (24 hours a day) from 5 July 2020 to 11 July 2020, and performance measures were determined.

Based on the Chi-square goodness of fit test, there is no evidence requiring rejection of the null hypothesis, which states that the distribution of patient arrivals follows a Poisson distribution with an arrival rate equal to 3.63 patients per hour, and there is no evidence to reject the null hypothesis which states that the distribution of service times follows an exponential distribution, with a $\mu = 0.418$ service rate per hour.

In analyzing the results presented in Tables 1 and 2, we observed that:

1- The distribution of patients' arrival in the emergency unit of the Asir governmental hospital followed a Poisson distribution pattern with an arrival rate of 3.63 patients per hour.
2- The distribution of service times for patients in the emergency unit of Asir governmental hospital followed an exponential distribution pattern with a $\mu = 0.418$ service rate per hour.
3- Services were provided to patients on a first-come, first-served basis.Thus, the appropriate mathematical model with which to study the queuing system is:

$$(M\backslash M\backslash\ 1)(FCFS\backslash\infty\backslash\infty)$$

The research aimed to build a computer simulation model, creating an honest representation of the reality of the system, to help decision-makers eliminate the waiting period. The model was built using the QSB program, The simulation results showed that the emergency unit suffers from a waiting phenomenon. Decision-makers must overcome this problem by adding service channels to minimize the wait time of patients and to increase the efficiency of the emergency unit. When analyzing the results shown in Table 4, we observed that the traffic intensity was 99.98%, which indicates overcrowding of potential patients in the emergency unit of Asir governmental hospital. This represents an inadequate service system for the emergency unit. Hence, we recommend adding additional service channels. The expected number of patients in the queue is 1,582 patients, a large number. The expected number of patients in the system, being those in the queue plus those being served, is 1,583 patients, again a large number. The expected wait time for the patients in the queue before they are provided with the service, they are waiting for is to 448 hours, which indicates that the emergency unit is suffering from a severe waiting phenomenon. The decision-maker should study this phenomenon and seek to reduce the wait time for patients and increase the efficiency of the emergency unit to the greatest extent possible, as the current expected wait time for the patients in the system is equal to 451 hours. Table 5 shows four scenarios, where 5, 9, and 12 additional service channels are added to the system (C=1, C=6, C=10, C=13). The fourth scenario is clearly preferable to the first, second, and third scenarios because when 12 service channels are added, the traffic intensity decreases from 99.98% to 67.12%, which indicates a decrease in overcrowding of potential patients in the emergency unit, the probability of the system being idle is 32.88%, and the wait time reduced to 0.07 hours. From the above results, the recommended solution is fourth scenario as all performance measures supporting its application.

## 6. Conclusion

In this article, we applied queuing theory to solve the problem of overcrowding of potential patients in the emergency unit of Asir governmental hospital located in Abha city, Saudi Arabia, which affects patient wait time, the level of service provided, the quality of service, and the level of pressure on medical staff, in order to reduce the wait time for patients and to increase the efficiency of the emergency unit.

In this study, actual data was applied. Hourly data was collected from the emergency unit; patients' arrival time and service time were recorded for 7 working days (24 hours a day) from 5 July 2020 to 11 July 2020, and performance measures were determined. It was concluded that the distribution of patients' arrival in the emergency unit followed a Poisson distribution with an arrival rate of 3.63 patients per hour. The distribution of service time of patients followed an exponential distribution with $\mu = 0.418$ service per hour. Services were provided to patients on a first-come, first-served basis and the number of service channels assumed in the study was (C=1). Thus, the appropriate mathematical model with which to study the system was judged to be $(M\backslash M\backslash 1)(FCFS\backslash\infty\backslash\infty)$.

The simulation results indicate that decision-makers should add 12 further service channels to the system to overcome the waiting phenomenon and minimize the wait time of patients and increase the efficiency of the emergency unit. The proposed queuing model for this system is $(M\backslash M\backslash\ 12)(FCFS\backslash\infty\backslash\infty)$.

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