MAXIMUM LIKELIHOOD ESTIMATION OF THE FLEXIBLE WEIBULL DISTRIBUTION PARAMETERS BASED ON TYPE I CENSORED DATA

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Abstract

This article discusses the estimation of parameters of the flexible Weibull (FW) distribution using the ML method based on type I censored sample.The Monte Carlo method is used to generate samples of different sizes from this distribution.The bias,MSE,the sample information matrix and the distribution of the estimates are also obtained. Concluding remarks regarding the results are provided.

Key Words : Maximum likelihood estimation; Reliability; Type-I censoring; Variance covariance matrix.

1. Introduction

Singh et al. (2005a) discussed the point estimation of the exponentiated- Weibull distribution parameters when all the three parameters of the distribution are unknown.. Waleed (2007)derived the maximum likelihood estimates of unknown parameters of the exponentiated- Weibull distribution based on type I and type II progressive interval censoring with fixed and random removals. Also the approximate asymptotic variance covariance matrix of the estimates were derived. The formulas to compute the expected duration and expected length of time under different schemes were given.

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The distribution function of the basic two parameter Weibull distribution is given by:

$$F(t) = 1 - e^{-\left(\frac{t}{\beta}\right)^{\alpha}} , \qquad t > 0 ; \qquad \alpha, \beta > 0 \tag{1.1}$$

This distribution is one of, the most popular life-testing distributions. It is of utmost interest to both theoretical statisticians and practitioners. Its density function changes rapidly as the shape parameter α changes from values less than one to values greater than one. It is a generalization of the exponential distribution and a special case of the extreme value distribution [see Johnson et al (1994)]. Weibull distributions with $\alpha < 1$ have a failure rate that decreases with time. When α is close or equal to one it will have a fairly constant failure rate, and when $\alpha > 1$ it will have a failure rate that increases with time. However, over many years and across a wide variety of mechanical and electronic systems, practitioners have calculated empirical population failure rate as units of age and repeatedly obtained graphs of failure rates that exhibit all three cases mentioned above. Because of the shape of the hazard rate function with the three characteristics in sequence with the decreasing failure rate in early life followed by a fairly constant failure rate in midlife, and then an increasing failure rate, it has become known as the "bathtub failure rate. Researchers for most part have tried to stay close to the Weibull distributions. Many generalizations and modifications of the Weibull distributions have been introduced to give bathtub failure rate [Xie et al. (2002)].

Gurvich et al. (1997) introduced a class of distributions characterized by the cumulative distribution function:

$$F(t) = 1 - \exp\{- a\, G(t)\} \qquad , \qquad t > 0 \tag{1.2}$$

with parameter a > 0, where G(t) is a monotonically increasing function of t, G(t) > 0. They considered (1.2) as a generalized Weibull distribution and when G(t) takes different formulas, different distributions can be obtained.

Lai et al, (2003) reviewed several families of extended Weibull distributions, while a comprehensive taxonomy of Weibull models can be found in Murthy et al. (2004). They discussed additional applications, and gave a methodological review of the "Weibull area". They also suggested further study of various Weibull- type distributions and model selection. Nadarajah and Kotz (2005) showed that several existing life distributions may be expressed in the form (1.2). Another modification of the Weibull distribution was introduced by Bebbington et al. (2006); They discussed an extension of the Weibull distribution. This distribution is another member of the Weibull family, which they named the flexible Weibull (FW)distribution. It does not have a monotonic failure rate function of t, but it is the class of lifetime distribution which has a bathtub shaped failure rate function. It is very important because the lifetime of electronic, electromechanical, and mechanical products are often modeled with this class. This distribution includes the bathtub shape, the modified bathtub shape and other different shapes for failure rate. So it is quite flexible.Amira et al. (2010) discussed the estimation of the FW distribution parameters based on type II censored sample using the maximum likelihood method.

Let t be a lifetime random variable with a density function given by:

$$f(t,\alpha,\beta) = \left(\alpha + \frac{\beta}{t^2}\right) \exp\left(\alpha t - \frac{\beta}{t}\right) \exp\left(-e^{\alpha t - \frac{\beta}{t}}\right), \; t > 0 \; ; \; \alpha, \; \beta > 0 \quad (1.3)$$

and a cumulative distribution function given by:

$$F(t) = 1 - \exp\left(-e^{\alpha t - \frac{\beta}{t}}\right), \; t > 0 \quad (1.4)$$

The reliability function and the failure rate function of this two – parameter flexible Weibull distribution are given respectively by:

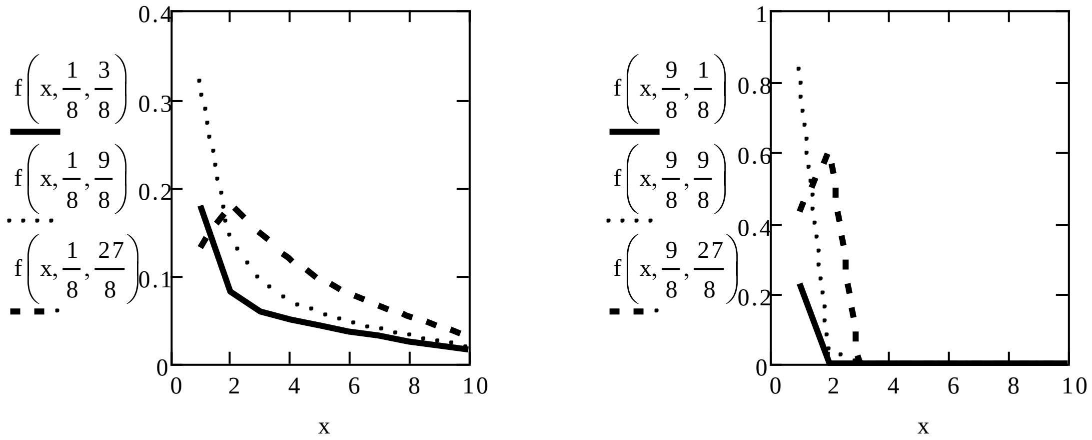
$$R(t) = \exp\left(-e^{\alpha t - \frac{\beta}{t}}\right), \quad t > 0 ; \; \alpha, \beta > 0, \quad (1.5)$$

$$h(t) = \left(\alpha + \frac{\beta}{t^2}\right) \exp\left(\alpha t - \frac{\beta}{t}\right), \; t > 0 ; \; \alpha, \; \beta > 0 \quad (1.6)$$

The formulas for the mean and the variance are difficult to obtain in closed form, the quantiles are easy to evaluate, Let t_p be the pth quantile of t. By considering the log-log transformation of p=R(t), we have log (-log p) = (αt^2 - β) / t, and so t_p is a solution of the quadratic equation αt^2 – log (- log p) t - β = 0. Since the solutions have to be non – negative, the only solution is:

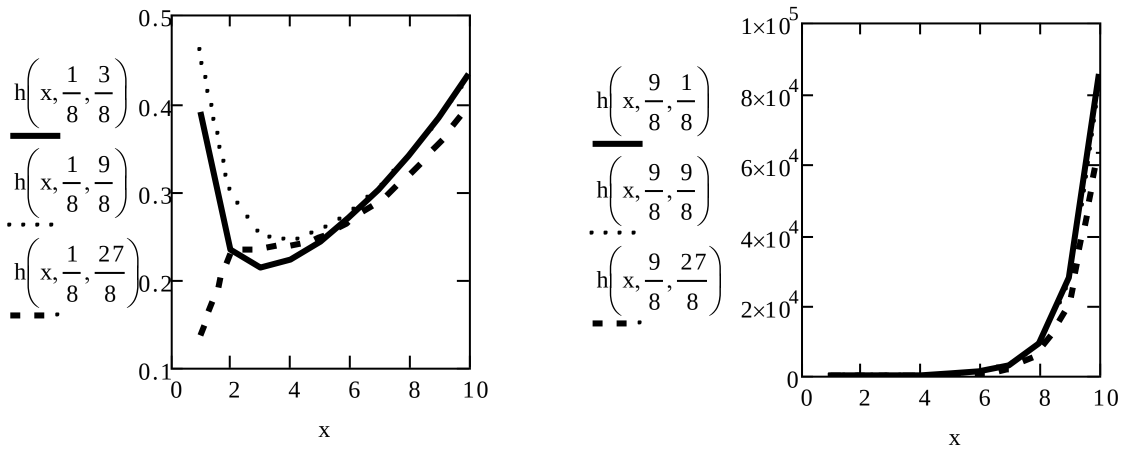
$$t_p = \frac{1}{2\alpha}\left(\log\left(-\log p\right) + \sqrt{\{\log\left(-\log p\right)\}^2 + 4\alpha\beta} \right)$$

The curves of the pdf and h (t) of the flexible Weibull distributions with parameters α, β are plotted in Figure (1) and Figure (2).



α = 1/8, β = 3/8, 9/8, 27/8 α = 9/8, β= 1/8, 9/8, 27/8

Figure (1): The Probability Density Function



α = 1/8, β = 3/8, 9/8, 27/8 α = 9/8, β= 1/8, 9/8, 27/8

Figure (2): The Faillure Rate Funcrtion

The shapes of the density and failure rate functions are illustrated for selected values of α and β in Figure (1) and Figure (2).. We see that as β decreases, the failure rate function becomes more bathtub-like;, while, as α increase, the bathtub' becomes shallower. Bebbington et al. (2006) showed that the failure rate function h (t) is increasing if and only if α β > 27/64, when α β < 27/64, the distribution is modified bathtub shape (MBT).

The maximum likelihood estimate for the parameters for type I censored sample data are obtained in section 2. In section 3, Monte Carlo simulation method is used, and concluding remarks are given.

2. Maximum Likelihood Estimates Based on Type I Censoring

If the experiment is to be terminated at time T, then the lifetimes can be known exactly only for these items that fail before time T. let k be the number of items that fail before time T. Data obtained under type I censoring can be represented by the n pairs of random variables (t_i, δ_i), I = 1, 2,…, n where

$$\delta_i = \begin{cases} 1 & \text{if} \quad t_i \le T_i \\ 0 & if \quad t_i > T_i \end{cases}$$

where $\sum_{i=1}^{n} \delta_i = k$, and t_i is the lifetime of the i-th item. δ_i is an indicator of whether or not the observed value t_i is censored. Given these n pairs, the likelihood function is

$$L(\alpha,\ \beta,\ t) = \frac{n!}{(n-k)} \prod_{i=1}^{n} f(t_i)[1 - F(T)]^{n-k}\ , \tag{2.1}$$

where f(t) and F(t) are given by Equations (1.3) and (1.4) respectively. Substituting (1.3) and (1.4) in (2.1), one obtains the following likelihood function:

$$L(\alpha,\ \beta,\ t) = \frac{n!}{(n-k)} \prod_{i=1}^{k} \left(\alpha + \frac{\beta}{t_i^2} \right) \exp\left(\alpha \sum_{i=1}^{k} t_i - \beta \sum_{i=1}^{k} \frac{1}{t_i} \right) \exp\left(-\sum_{i=1}^{k} e^{\alpha t_i} - \frac{\beta}{t_i} \right) \exp\left(-e^{\alpha T - \frac{\beta}{T}} \right)^{n-k}$$

The natural logarithm of the above function is given by:

l = 1n L (t, α, β)

$$l = \ln n! - \ln (n-k)! + \sum_{i=1}^{k} \ln\left(\alpha + \frac{\beta}{t_i^2}\right) + \alpha \sum_{i=1}^{k} t_i - \sum_{i=1}^{k} \frac{\beta}{t_i}$$

$$- \sum_{i=1}^{k} e^{\alpha t_i - \frac{\beta}{t_k}} - (n-k) e^{\alpha T - \frac{\beta}{T}} \qquad (2.2)$$

Differentiating (2.2) with respect to α and β and setting to zero, we obtain:

$$\left.\frac{\partial l}{\partial \alpha}\right|_{\hat{\alpha},\hat{\beta}} = \sum_{i=1}^{k} \frac{t_i^2}{\hat{\alpha} t_i^2 + \hat{\beta}} + \sum_{i=1}^{k} t_i - \sum_{i=1}^{k} t_i e^{\hat{\alpha} t_i - \frac{\hat{\beta}}{t_i}} - (n-k) T e^{\hat{\alpha} T - \frac{\hat{\beta}}{T}} = 0, \qquad (2.3)$$

$$\left.\frac{\partial l}{\partial \beta}\right|_{\hat{\alpha},\hat{\beta}} = \sum_{i=1}^{k} \frac{1}{\hat{\alpha} t_i^2 + \hat{\beta}} - \sum_{i=1}^{k} \frac{1}{t_i} + \sum_{i=1}^{k} \frac{1}{t_i} e^{\hat{\alpha} t_i - \frac{\hat{\beta}}{t_i}} + (n-k) \frac{1}{T} e^{\hat{\alpha} T - \frac{\hat{\beta}}{T}} = 0 \qquad (2.4)$$

The maximum likelihood estimates $\hat{a}$ and $\hat{\beta}$ can then be obtained by solving the two non linear equations (2.3), (2.4). The values of $\hat{a}$ and $\hat{\beta}$ will be obtained using Math Cad (14).The estimates of α and β are found in Tables 1,2,3,4 for different initial values of α and β.

The variance – covariance matrix of MLE's is given by the elements of the inverse of the Fisher information matrix.

$$I(\alpha, \beta) = -E \begin{pmatrix} \frac{\partial^2 I}{\partial \alpha^2} & \frac{\partial^2 I}{\partial \alpha \, \partial \beta} \\ \frac{\partial^2 I}{\partial \alpha \, \partial \beta} & \frac{\partial^2 I}{\partial \beta^2} \end{pmatrix} \qquad (2.5)$$

Ufortunately, the exact mathematical expressions for the above expectations are very difficult to obtain. Therefore, the observed information matrix can be estimated by dropping the expectation operator E. The result will be the estimated Fisher information matrix $\hat{I}\ (\hat{a}, \hat{\beta})$ in which $\hat{\alpha}$ and $\hat{\beta}$ are used instead of α and β [Cohen (1965)].

The sample information matrix is obtained in Tables 1 to 4, which is defined as follows

$$\hat{I}(\alpha, \beta) = - \begin{pmatrix} \frac{\partial^2 I}{\partial \alpha^2} & \frac{\partial^2 I}{\partial \alpha \, \partial \beta} \\ \frac{\partial^2 I}{\partial \alpha \, \partial \beta} & \frac{\partial^2 I}{\partial \beta^2} \end{pmatrix}_{\substack{\alpha = \hat{\alpha} \\ \beta = \hat{\beta}}} \qquad (2.6)$$

The estimated variance – covariance matrix is obtained by inverting the observed information matrix as follows

$$\hat{V} = [\hat{I}(\alpha, \beta)]^{-1} \qquad (2.7)$$

from which we can get $\hat{V}(\hat{\alpha})$, $\hat{V}(\hat{\beta})$ and $\hat{Cov}(\hat{\alpha}, \hat{\beta})$.

The elements of $[\hat{I}(\alpha, \beta)]$ in (2.6) will be obtained by differentiating (2.3) and (2.4) with respect to α and β

$$\frac{\partial^2 I}{\partial \alpha^2} = \sum_{i=1}^{k} \frac{-\left(t_i^2\right)^2}{\left(\alpha t_i^2 + \beta\right)^2} - \sum_{i=1}^{n} t_i^2\, e^{\alpha t_i - \frac{\beta}{t_i}} - (n-k) T^2\, e^{\alpha T - \frac{\beta}{T}} \qquad (2.8)$$

$$\frac{\partial^2 I}{\partial \alpha\, \partial \beta} = \sum_{i=1}^{k} \frac{-t_i^2}{\left(\alpha t_i^2 + \beta\right)^2} + \sum_{i=1}^{k} e^{\alpha t_i - \frac{\beta}{t_i}} + (n-k) e^{\alpha T - \frac{\beta}{T}} \qquad (2.9)$$

$$\frac{\partial^2 I}{\partial \beta^2} = \sum_{i=1}^{k} \frac{-1}{\left(\alpha t_i^2 + \beta\right)^2} - \sum_{i=1}^{k} \left(\frac{1}{t_i}\right)^2 e^{\alpha t_i - \frac{\beta}{t_i}} - (n-k) \left(\frac{1}{T}\right)^2 e^{\alpha T - \frac{\beta}{T}} \qquad (2.10)$$

Putting $\hat{a}$ and $\hat{\beta}$ in place of α and β,the elements of the sample information matrices obtained are found in Tables 1 to 4.

3- Sampling Distribution of the Maximum Likelihood Estimates

Is this section, the sampling distribution of the maximum likelihood estimators for different value of the parameters α and β are obtained. The Pearson's system approach is used for this purpose. The selection approach is based on computing the following criterion for fitting the distribution family.

$$K = \frac{\gamma_3 (\gamma_4 + 3)^2}{4(4\gamma_4 - 3\gamma_3)(2\gamma_4 - 3\gamma_3 - 6)} \qquad (3.1)$$

where γ_3 and γ_4 denote the skewness and kurtosis measures respectively. For different values of K there exist different types of

Pearson's distributions [Elderton and Johonson (1969), Johnson and Kotz (1995)].

The Pearson's system approach could be summarized in the following steps:

1- Calculate the first four central moments from the resulting MLE's of both α and β for each sample in type I censored samples.

2- Use the central moments to compute γ_3 and γ_4 for each estimator.

3- Use γ_3 and γ_4 to calculate K for $\hat{\alpha}$ and $\hat{\beta}$.

4- Select the appropriate distribution from Pearson's family according to the values of γ_3 , γ_4 , and K for each estemate as follows:

(a) If K < 0, select Pearson's type I distribution.

(b) If $\gamma_3 = 0$ and $\gamma_4 < 3$, select Pearson's type II distribution.

(c) If $K = \infty$ and $2\gamma_4 - 3\gamma_3 - 6 = 0$ select Pearson's type III distribution.

(d) If 0 < K < 1, select Pearson's type IV distribution.

(e) If K = 1, select Pearson's type V distribution.

(f) If K > 1, select Pearson's type VI distribution.

(g) If $\gamma_3 = 0$ and $\gamma_4 > 3$, select Pearson's type VII distribution.

To obtain the sample central moments for the estimates ,we use the following formula:

$$M_{r,n} = \frac{\sum \left[\hat{\theta}_{nj} - \hat{\theta}_{n} \right]^{r}}{N} \qquad (3.2)$$

where θ is the parameter to be estimated j= (1, …, N) number of repetitions,n=10,20,30,50,100 and r = 1,2,3,4,… the order of moment.

The coefficient of skewness is :

$$\gamma_3 = \frac{M_3}{M_2^{3/2}} \quad , \qquad (3.3)$$

and the coefficient of kurtosis:

$$\gamma_4 = \frac{M_4}{M_2^{2}} \quad , \qquad (3.4)$$

Depending on the values of γ_3, γ_4 , and K the sampling distributions of $\hat{\alpha}$ and $\hat{\beta}$ are dertermined,Tables 5,6,7,8.

4- Simulation Study and Concluding Remarks

The population parameters are estimated using the mathematical computing package Math Cad (14) to solve equations (2.3) and (2.4) to get the MLE's of the parameters α and β in type -I censoring .

The simulation results are obtained according to the following steps:

1- Select initial values of α, β.

2- Given α and β, generate random samples of size (n = 10, 20, 30, 50, 100),from the flexible Weibull distribution defined in equation (1-3), This can be achieved by generating random samples from the uniform (0,1) distribution. Then, the uniform random variables can be transformed to the flexible Weibull random variables using the following transformation,

$$1 - e^{-e^{\alpha t - \frac{\beta}{t}}} = u \qquad (4.1)$$

3- Sort the random samples to obtain the ordered samples.

4-Put the generated ordered sample observations in equations (2.3) and (2.4) to obtain $\hat{\alpha}$ and $\hat{\beta}$.

5-Repeat steps 2,3 and 4 N times ,where N=1000.

The maximum likelihood estimates of the parameters, the bias, the relative absolute bias(RAB), the mean square error(MSE), the relative mean square error(RMSE) , the variance, and the sample information matrix have been calculated,Table1,2,3,4.From these tables , it is observed that:

1- The estimates of the distribution parameters are very close to the true values of these parameters except for the case when n=10, and p=0.5. This is clear when we look at the relative obsolute bias (RAB) and the relative mean square error (RMSE). However,the estimates are getting better when the sample size increases.
2- Better estimates are obtained when the level of censoring decreased and this remark is expected since decreasing the level of censoring means that more information is provided by the sample and hence increases the accuracy of the estimates.
3- The MLE's are consistent. We observe that the MLE decreases when n increases for each parameter.

4- It is advised not to estimate the parameters α and β of this distribution in the case of small sample sizes [for example n=10and p=0.5]

5- Tables 5 to 8 present the sampling distribution of the MLE's of α and β according to Pearson's type of distribution.

6- From Tables 5 to 8 it is observed that the sampling distributions of the estimates $\hat{\alpha}$ and $\hat{\beta}$ are classified as follows:

P / D	$\hat{\alpha}$			$\hat{\beta}$		
Sampling distribution	**IV**	**I**	**VI**	**IV**	**I**	**VI**
Ratio	**35%**	**57%**	**0.08%**	**48%**	**44%**	**0.08%**

It is observed that 92% of the sampling distribution of $\hat{\alpha}$ and $\hat{\beta}$ are Pearson's type I and type IV, and of only 8% of these distributions are Pearson's type VI.

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Table (1): Estimates, Bias, Relative Absolute Bias(RAB),Variance,(MSE),(RMSE) and Sample Information Matrix of The MLEs Based on type I Censored Sample, at $\alpha = 1/8$, $\beta = 3/8$, n=10,20,30,50,100

n	P	Estimate	Bias	R A B	VAR	MSE	RME	S. Infor.Matrix
10	0.8	$\hat{\alpha}$ = 0.129	4.404×10^{-3}	0.035	0.020	0.020	0.160	$\begin{pmatrix} 6.842\times10^{-3} & -1.005\times10^{-3} \\ -1.005\times10^{-3} & 0.027 \end{pmatrix}$
		$\hat{\beta}$ = 0.455	0.080	0.212	0.069	0.075	0.200	
	0.7	$\hat{\alpha}$ = 0.103	-0.022	0.174	0.044	0.045	0.358	$\begin{pmatrix} 0.019 & -2.644\times10^{-3} \\ -2.644\times10^{-3} & 0.026 \end{pmatrix}$
		$\hat{\beta}$ = 0.453	0.078	0.209	0.064	0.070	0.187	
	0.5	$\hat{\alpha}$ = 0.073	-0.052	0.418	1.174	1.177	9.414	$\begin{pmatrix} 0.375 & 0.019 \\ 0.019 & 0.037 \end{pmatrix}$
		$\hat{\beta}$ = 0.534	0.159	0.425	3.546	3.571	9.523	
20	0.8	$\hat{\alpha}$ = 0.129	4.197×10^{-3}	0.034	5.24×10^{-3}	5.258×10^{-3}	0.042	$\begin{pmatrix} 3.39\times10^{-3} & -5.225\times10^{-4} \\ -5.225\times10^{-4} & 0.011 \end{pmatrix}$
		$\hat{\beta}$ = 0.410	0.035	0.094	0.014	0.016	0.042	
	0.7	$\hat{\alpha}$ = 0.121	-3.92×10^{-3}	0.031	0.016	0.016	0.126	$\begin{pmatrix} 9.314\times10^{-3} & -1.131\times10^{-3} \\ -1.131\times10^{-3} & 0.011 \end{pmatrix}$
		$\hat{\beta}$ = 0.411	0.036	0.096	0.014	0.016	0.042	
	0.5	$\hat{\alpha}$ = 0.126	8.229×10^{-4}	6.583×10^{-3}	0.221	0.221	1.766	$\begin{pmatrix} 0.140 & 2.927\times10^{-3} \\ 2.927\times10^{-3} & 0.011 \end{pmatrix}$
		$\hat{\beta}$ = 0.411	0.036	0.095	0.016	0.017	0.046	
30	0.8	$\hat{\alpha}$ = 0.129	3.839×10^{-3}	0.031	2.809×10^{-3}	2.824×10^{-3}	0.023	$\begin{pmatrix} 2.215\times10^{-3} & -3.562\times10^{-4} \\ -3.562\times10^{-4} & 6.903\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.394	0.019	0.050	8.034×10^{-3}	8.389×10^{-3}	0.022	
	0.7	$\hat{\alpha}$ = 0.129	3.671×10^{-3}	0.029	7.322×10^{-3}	7.335×10^{-3}	0.059	$\begin{pmatrix} 6.022\times10^{-3} & -7.071\times10^{-4} \\ -7.071\times10^{-4} & 6.944\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.394	0.019	0.051	8.089×10^{-3}	8.454×10^{-3}	0.023	
	0.5	$\hat{\alpha}$ = 0.123	-2.031×10^{-3}	0.016	0.098	0.098	0.781	$\begin{pmatrix} 0.090 & 1.541\times10^{-3} \\ 1.541\times10^{-3} & 7.027\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.394	0.019	0.051	7.993×10^{-3}	8.36×10^{-3}	0.022	

To be continued

n	P	Estimate	Bias	RAB	VAR	MSE	RMSE	S. Infor.matrix
50	0.8	$\hat{\alpha} = 0.128$	3.457×10^{-3}	0.028	1.396×10^{-3}	1.408×10^{-3}	0.011	$\begin{pmatrix} 1.326\times10^{-3} & -2.157\times10^{-4} \\ -2.157\times10^{-4} & 4.003\times10^{-3} \end{pmatrix}$
		$\hat{\beta} = 0.388$	0.013	0.033	4.336×10^{-3}	4.493×10^{-3}	0.012	
	0.7	$\hat{\alpha} = 0.127$	2.31×10^{-3}	0.018	3.816×10^{-3}	3.821×10^{-3}	0.031	$\begin{pmatrix} 3.638\times10^{-3} & -4.373\times10^{-4} \\ -4.373\times10^{-4} & 4.026\times10^{-3} \end{pmatrix}$
		$\hat{\beta} = 0.388$	0.013	0.034	4.341×10^{-3}	4.504×10^{-3}	0.012	
	0.5	$\hat{\alpha} = 0.120$	-4.674×10^{-3}	0.037	0.059	0.059	0.475	$\begin{pmatrix} 0.054 & 8.223\times10^{-4} \\ 8.223\times10^{-4} & 4.079\times10^{-3} \end{pmatrix}$
		$\hat{\beta} = 0.387$	0.012	0.033	4.37×10^{-3}	4.524×10^{-3}	0.012	
100	0.8	$\hat{\alpha} = 0.126$	1.365×10^{-3}	0.011	7.078×10^{-4}	7.097×10^{-4}	5.677×10^{-3}	$\begin{pmatrix} 6.511\times10^{-4} & -1.085\times10^{-4} \\ -1.085\times10^{-4} & 1.949\times10^{-3} \end{pmatrix}$
		$\hat{\beta} = 0.381$	6.092×10^{-3}	0.016	1.967×10^{-3}	2.004×10^{-3}	5.345×10^{-3}	
	0.7	$\hat{\alpha} = 0.124$	-1.452×10^{-3}	0.012	1.87×10^{-3}	1.872×10^{-3}	0.015	$\begin{pmatrix} 1.787\times10^{-3} & -2.228\times10^{-4} \\ -2.228\times10^{-4} & 1.96\times10^{-3} \end{pmatrix}$
		$\hat{\beta} = 0.381$	6.478×10^{-3}	0.017	1.978×10^{-3}	2.02×10^{-3}	5.387×10^{-3}	
	0.5	$\hat{\alpha} = 0.123$	-2.386×10^{-3}	0.019	0.026	0.026	0.207	$\begin{pmatrix} 0.026 & 3.649\times10^{-4} \\ 3.649\times10^{-4} & 1.988\times10^{-3} \end{pmatrix}$
		$\hat{\beta} = 0.382$	6.536×10^{-3}	0.017	1.976×10^{-3}	2.019×10^{-3}	5.384×10^{-3}	

Table (2):Estimates,Bias,Relative Absolute Bias(RAB),Variance,(MSE),(RMSE) and Sample Information Matrix of the MLEs Based on type I Censored Sample, at $\alpha = 1/2$, $\beta = 9/8$, n=10,20,30,50,100

n	P	Estimate	Bias	RAB	VAR	MSE	RMSE	S. Infor.matrix
10	0.8	$\hat{\alpha}$ = 0.556	0.056	0.112	0.065	0.069	0.137	$\begin{pmatrix} 0.045 & 0.042 \\ 0.042 & 0.192 \end{pmatrix}$
		$\hat{\beta}$ = 1.307	0.182	0.162	0.314	0.348	0.309	
	0.7	$\hat{\alpha}$ = 0.560	0.06	0.121	0.093	0.096	0.193	$\begin{pmatrix} 0.075 & 0.060 \\ 0.060 & 0.204 \end{pmatrix}$
		$\hat{\beta}$ = 1.318	0.193	0.171	0.359	0.396	0.352	
	0.5	$\hat{\alpha}$ = 0.678	0.178	0.356	2.517	2.549	5.098	$\begin{pmatrix} 0.31 & 0.209 \\ 0.209 & 0.34 \end{pmatrix}$
		$\hat{\beta}$ = 1.568	0.443	0.394	11.867	12.064	10.723	
20	0.8	$\hat{\alpha}$ = 0.534	0.034	0.068	0.027	0.028	0.055	$\begin{pmatrix} 0.022 & 0.017 \\ 0.017 & 0.084 \end{pmatrix}$
		$\hat{\beta}$ = 1.214	0.089	0.079	0.098	0.106	0.094	
	0.7	$\hat{\alpha}$ = 0.538	0.038	0.067	0.042	0.044	0.087	$\begin{pmatrix} 0.035 & 0.025 \\ 0.025 & 0.088 \end{pmatrix}$
		$\hat{\beta}$ = 1.219	0.094	0.084	0.111	0.119	0.106	
	0.5	$\hat{\alpha}$ = 0.574	0.074	0.148	0.124	0.130	0.259	$\begin{pmatrix} 0.110 & 0.060 \\ 0.060 & 0.106 \end{pmatrix}$
		$\hat{\beta}$ = 1.242	0.117	0.104	0.153	0.166	0.148	
30	0.8	$\hat{\alpha}$ = 0.519	0.019	0.038	0.016	0.017	0.033	$\begin{pmatrix} 0.014 & 0.011 \\ 0.011 & 0.053 \end{pmatrix}$
		$\hat{\beta}$ = 1.176	0.051	0.046	0.059	0.062	0.055	
	0.7	$\hat{\alpha}$ = 0.521	0.021	0.042	0.025	0.025	0.050	$\begin{pmatrix} 0.023 & 0.015 \\ 0.015 & 0.055 \end{pmatrix}$
		$\hat{\beta}$ = 1.178	0.053	0.047	0.061	0.064	0.057	
	0.5	$\hat{\alpha}$ = 0.526	0.026	0.052	0.070	0.070	0.140	$\begin{pmatrix} 0.069 & 0.035 \\ 0.035 & 0.064 \end{pmatrix}$
		$\hat{\beta}$ = 1.183	0.058	0.051	0.071	0.075	0.066	

To be continued

N	P	Estimate	Bias	RAB	VAR	MSE	RMSE	S. Infor.matrix
50	0.8	$\hat{\alpha}$ = 0.517	0.017	0.034	8.971×10^{-3}	9.254×10^{-3}	0.019	$\begin{pmatrix} 8.473\times10^{-3} & 6.237\times10^{-3} \\ 6.237\times10^{-3} & 0.031 \end{pmatrix}$
		$\hat{\beta}$ = 1.159	0.034	0.030	0.033	0.034	0.030	
	0.7	$\hat{\alpha}$ = 0.517	0.017	0.034	0.014	0.014	0.029	$\begin{pmatrix} 0.013 & 8.819\times10^{-3} \\ 8.819\times10^{-3} & 0.032 \end{pmatrix}$
		$\hat{\beta}$ = 1.160	0.035	0.031	0.034	0.036	0.032	
	0.5	$\hat{\alpha}$ = 0.518	0.018	0.036	0.043	0.043	0.086	$\begin{pmatrix} 0.041 & 0.020 \\ 0.020 & 0.037 \end{pmatrix}$
		$\hat{\beta}$ = 1.163	0.038	0.034	0.040	0.042	0.037	
100	0.8	$\hat{\alpha}$ = 0.507	6.517×10^{-3}	0.013	4.428×10^{-3}	4.47×10^{-3}	8.941×10^{-3}	$\begin{pmatrix} 4.19\times10^{-3} & 2.994\times10^{-3} \\ 2.994\times10^{-3} & 0.015 \end{pmatrix}$
		$\hat{\beta}$ = 1.143	0.018	0.016	0.015	0.015	0.014	
	0.7	$\hat{\alpha}$ =0.504	3.545×10^{-3}	7.09×10^{-3}	6.947×10^{-3}	6.96×10^{-3}	0.014	$\begin{pmatrix} 6.633\times10^{-3} & 4.212\times10^{-3} \\ 4.212\times10^{-3} & 0.016 \end{pmatrix}$
		$\hat{\beta}$ = 1.142	0.017	0.015	0.016	0.016	0.014	
	0.5	$\hat{\alpha}$ = 0.509	9.215×10^{-3}	0.018	0.019	0.019	0.038	$\begin{pmatrix} 0.020 & 9.82\times10^{-3} \\ 9.82\times10^{-3} & 0.018 \end{pmatrix}$
		$\hat{\beta}$ = 1.145	0.020	0.018	0.018	0.018	0.016	

Table (3):Estimates,Bias,Relative Absolute Bias(RAB),Variance,(MSE),(RMSE) and Sample Information Matrix of the MLEs Based on type I Censored Sample, at $\alpha = 9/8$, $\beta = 9/8$, n=10,20,30,50,100

N	P	Estimate	Bias	RAB	VAR	MSE	RMSE	S. Infor.matrix
10	0.8	$\hat{\alpha}$ = 1.306	0.181	0.161	0.268	0.301	0.268	$\begin{pmatrix} 0.183 & 0.124 \\ 0.124 & 0.180 \end{pmatrix}$
		$\hat{\beta}$ = 1.317	0.192	0.170	0.241	0.278	0.247	
	0.7	$\hat{\alpha}$ = 1.337	0.212	0.189	0.401	0.446	0.397	$\begin{pmatrix} 0.281 & 0.170 \\ 0.170 & 0.203 \end{pmatrix}$
		$\hat{\beta}$ = 1.343	0.218	0.193	0.320	0.368	0.327	
	0.5	$\hat{\alpha}$ = 1.535	0.410	0.364	1.612	1.780	1.582	$\begin{pmatrix} 0.815 & 0.388 \\ 0.388 & 0.299 \end{pmatrix}$
		$\hat{\beta}$ = 1.452	0.327	0.290	0.881	0.987	0.878	
20	0.8	$\hat{\alpha}$ = 1.225	0.100	0.089	0.100	0.110	0.098	$\begin{pmatrix} 0.085 & 0.053 \\ 0.053 & 0.079 \end{pmatrix}$
		$\hat{\beta}$ = 1.221	0.096	0.085	0.101	0.111	0.098	
	0.7	$\hat{\alpha}$ = 1.233	0.108	0.096	0.145	0.157	0.139	$\begin{pmatrix} 0.125 & 0.070 \\ 0.070 & 0.086 \end{pmatrix}$
		$\hat{\beta}$ = 1.227	0.102	0.091	0.114	0.125	0.111	
	0.5	$\hat{\alpha}$ = 1.288	0.163	0.145	0.864	0.890	0.791	$\begin{pmatrix} 0.325 & 0.141 \\ 0.141 & 0.112 \end{pmatrix}$
		$\hat{\beta}$ = 1.26	0.135	0.120	0.453	0.471	0.419	
30	0.8	$\hat{\alpha}$ = 1.176	0.051	0.045	0.060	0.063	0.056	$\begin{pmatrix} 0.054 & 0.032 \\ 0.032 & 0.048 \end{pmatrix}$
		$\hat{\beta}$ = 1.169	0.044	0.039	0.052	0.054	0.048	
	0.7	$\hat{\alpha}$ = 1.182	0.057	0.050	0.083	0.086	0.076	$\begin{pmatrix} 0.08 & 0.042 \\ 0.042 & 0.052 \end{pmatrix}$
		$\hat{\beta}$ = 1.172	0.047	0.042	0.054	0.057	0.050	
	0.5	$\hat{\alpha}$ = 1.226	0.101	0.089	0.206	0.216	0.192	$\begin{pmatrix} 0.197 & 0.082 \\ 0.082 & 0.066 \end{pmatrix}$
		$\hat{\beta}$ = 1.191	0.066	0.058	0.074	0.078	0.070	

To be continued

n	P	Estimate	Bias	RAB	VAR	MSE	RMSE	S. Infor.matrix
50	0.8	$\hat{\alpha}$ = 1.146	0.021	0.019	0.031	0.032	0.028	$\begin{pmatrix} 0.032 & 0.019 \\ 0.019 & 0.029 \end{pmatrix}$
		$\hat{\beta}$ = 1.153	0.028	0.025	0.030	0.031	0.028	
	0.7	$\hat{\alpha}$ = 1.143	0.018	0.016	0.046	0.046	0.041	$\begin{pmatrix} 0.047 & 0.025 \\ 0.025 & 0.031 \end{pmatrix}$
		$\hat{\beta}$ = 1.153	0.028	0.025	0.033	0.034	0.030	
	0.5	$\hat{\alpha}$ = 1.165	0.040	0.036	0.131	0.132	0.118	$\begin{pmatrix} 0.115 & 0.047 \\ 0.047 & 0.038 \end{pmatrix}$
		$\hat{\beta}$ = 1.163	0.038	0.034	0.045	0.046	0.041	
100	0.8	$\hat{\alpha}$ = 1.134	8.688×10^{-3}	7.722×10^{-3}	0.015	0.015	0.014	$\begin{pmatrix} 0.016 & 9.16\times10^{-3} \\ 9.16\times10^{-3} & 0.014 \end{pmatrix}$
		$\hat{\beta}$ = 1.136	0.011	0.010	0.014	0.015	0.013	
	0.7	$\hat{\alpha}$ = 1.138	0.013	0.011	0.022	0.022	0.019	$\begin{pmatrix} 0.023 & 0.012 \\ 0.012 & 0.015 \end{pmatrix}$
		$\hat{\beta}$ = 1.138	0.013	0.012	0.015	0.016	0.014	
	0.5	$\hat{\alpha}$ = 1.145	0.020	0.018	0.059	0.059	0.053	$\begin{pmatrix} 0.056 & 0.022 \\ 0.022 & 0.018 \end{pmatrix}$
		$\hat{\beta}$ = 1.142	0.017	0.015	0.021	0.021	0.019	

Table (4):Estimates, Bias, Relative Absolute Bias(RAB),Variance,(MSE),(RMSE) and Sample Information Matrix of The MLEs Based on type I Censored Sample, at $\alpha = 9/8$, $\beta = 1/2$, n=10,20,30,50,100

n	P	Estimate	Bias	RAB	VAR	MSE	RMSE	S. Infor.Matrix
10	0.8	$\hat{\alpha}$ = 1.279	0.154	0.137	0.364	0.388	0.345	$\begin{pmatrix} 0.229 & 0.041 \\ 0.041 & 0.037 \end{pmatrix}$
		$\hat{\beta}$ = 0.576	0.076	0.152	0.044	0.050	0.100	
	0.7	$\hat{\alpha}$ = 1.280	0.155	0.138	0.630	0.654	0.582	$\begin{pmatrix} 0.378 & 0.059 \\ 0.059 & 0.039 \end{pmatrix}$
		$\hat{\beta}$ = 0.581	0.081	0.162	0.056	0.063	0.125	
	0.5	$\hat{\alpha}$ = 1.450	0.325	0.289	2.218	2.324	2.066	$\begin{pmatrix} 1.513 & 0.190 \\ 0.190 & 0.063 \end{pmatrix}$
		$\hat{\beta}$ = 0.687	0.187	0.373	3.052	3.087	6.173	
20	0.8	$\hat{\alpha}$ = 1.181	0.056	0.050	0.133	0.136	0.121	$\begin{pmatrix} 0.109 & 0.017 \\ 0.017 & 0.016 \end{pmatrix}$
		$\hat{\beta}$ = 0.535	0.035	0.071	0.019	0.020	0.040	
	0.7	$\hat{\alpha}$ = 1.184	0.059	0.052	0.201	0.205	0.182	$\begin{pmatrix} 0.175 & 0.024 \\ 0.024 & 0.017 \end{pmatrix}$
		$\hat{\beta}$ = 0.536	0.036	0.073	0.020	0.022	0.043	
	0.5	$\hat{\alpha}$ = 1.199	0.074	0.066	0.667	0.673	0.598	$\begin{pmatrix} 0.544 & 0.057 \\ 0.057 & 0.02 \end{pmatrix}$
		$\hat{\beta}$ = 0.541	0.041	0.083	0.027	0.028	0.057	
30	0.8	$\hat{\alpha}$ = 1.182	0.057	0.051	0.076	0.079	0.070	$\begin{pmatrix} 0.072 & 0.011 \\ 0.011 & 0.01 \end{pmatrix}$
		$\hat{\beta}$ = 0.522	0.022	0.043	0.011	0.012	0.023	
	0.7	$\hat{\alpha}$ = 1.182	0.057	0.051	0.122	0.125	0.111	$\begin{pmatrix} 0.114 & 0.015 \\ 0.015 & 0.011 \end{pmatrix}$
		$\hat{\beta}$ = 0.522	0.022	0.045	0.012	0.013	0.025	
	0.5	$\hat{\alpha}$ = 1.182	0.057	0.051	0.380	0.383	0.341	$\begin{pmatrix} 0.348 & 0.035 \\ 0.035 & 0.013 \end{pmatrix}$
		$\hat{\beta}$ = 0.524	0.024	0.048	0.015	0.015	0.031	

To be continued

n	P	Estimate	Bias	RAB	VAR	MSE	RMSE	S. Infor.Matrix
50	0 .8	$\hat{\alpha}$ = 1.150	0.025	0.022	0.043	0.043	0.039	$\begin{pmatrix} 0.049 & 6.198\times10^{-3} \\ 6.198\times10^{-3} & 6.109\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.515	0.015	0.030	6.001×10^{-3}	6.232×10^{-3}	0.012	
	0.7	$\hat{\alpha}$ = 1.150	0.025	0.022	0.067	0.067	0.060	$\begin{pmatrix} 0.068 & 8.776\times10^{-3} \\ 8.776\times10^{-3} & 6.374\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.516	0.016	0.031	6.397×10^{-3}	6.638×10^{-3}	0.013	
	0.5	$\hat{\alpha}$ = 1.174	0.049	0.044	0.213	0.216	0.192	$\begin{pmatrix} 0.206 & 0.021 \\ 0.021 & 7.424\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.519	0.019	0.037	7.724×10^{-3}	8.072×10^{-3}	0.016	
100	0.8	$\hat{\alpha}$ = 1.146	0.021	0.019	0.023	0.023	0.020	$\begin{pmatrix} 0.021 & 3.019\times10^{-3} \\ 3.019\times10^{-3} & 2.994\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.509	9.341×10^{-3}	0.019	3.037×10^{-3}	3.125×10^{-3}	6.249×10^{-3}	
	0.7	$\hat{\alpha}$ = 1.144	0.019	0.017	0.034	0.034	0.031	$\begin{pmatrix} 0.034 & 4.259\times10^{-3} \\ 4.259\times10^{-3} & 3.117\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.509	9.175×10^{-3}	0.018	3.149×10^{-3}	3.233×10^{-3}	6.467×10^{-3}	
	0.5	$\hat{\alpha}$ = 1.154	0.029	0.026	0.098	0.099	0.088	$\begin{pmatrix} 0.101 & 9.904\times10^{-3} \\ 9.904\times10^{-3} & 3.598\times10^{-3} \end{pmatrix}$
		$\hat{\beta}$ = 0.510	0.010	0.021	3.567×10^{-3}	3.674×10^{-3}	7.347×10^{-3}	

Table (5)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution $\alpha = 1/8$, $\beta = 3/8$

N	P	Estimate	γ_3	γ_4	K	Dist^n
10	0.8	$\hat{\alpha} = 0.129$	0.802	11.158	0.068	IV
		$\hat{\beta} = 0.455$	4.161	32.652	0.239	IV
	0.7	$\hat{\alpha} = 0.103$	-0.586	6.158	-0.058	I
		$\hat{\beta} = 0.453$	4.027	32.599	0.229	IV
	0.5	$\hat{\alpha} = 0.073$	6.634	112.665	0.258	IV
		$\hat{\beta} = 0.534$	27.418	810.437	0.937	IV
20	0.8	$\hat{\alpha} = 0.129$	1.092	10.412	0.111	IV
		$\hat{\beta} = 0.410$	1.579	7.474	0.409	IV
	0.7	$\hat{\alpha} = 0.121$	-0.540	6.937	-0.048	I
		$\hat{\beta} = 0.411$	1.620	7.803	0.378	IV
	0.5	$\hat{\alpha} = 0.126$	-2.052	22.900	-0.077	I
		$\hat{\beta} = 0.411$	2.261	16.633	0.178	IV
30	0.8	$\hat{\alpha} = 0.129$	0.511	4.334	0.383	IV
		$\hat{\beta} = 0.394$	0.952	4.278	-2.956	I
	0.7	$\hat{\alpha} = 0.129$	-0.033	3.889	-0.013	I
		$\hat{\beta} = 0.394$	0.962	4.273	-2.632	I
	0.5	$\hat{\alpha} = 0.123$	-0.276	3.465	-0.112	I
		$\hat{\beta} = 0.394$	0.957	4.396	-11.345	I

50	0.8	$\hat{\alpha}$ = 0.128	0.333	3.281	-0.620	I
		$\hat{\beta}$ = 0.388	0.723	4.009	-4.281	I
	0.7	$\hat{\alpha}$ = 0.127	0.156	3.212	-2.908	I
		$\hat{\beta}$ = 0.388	0.722	3.992	-3.509	I
	0.5	$\hat{\alpha}$ = 0.120	-0.115	3.705	-0.049	I
		$\hat{\beta}$ = 0.387	0.756	4.027	-3.131	I
100	0.8	$\hat{\alpha}$ = 0.126	0.206	3.383	1.107	VI
		$\hat{\beta}$ = 0.381	0.768	4.574	0.814	IV
	0.7	$\hat{\alpha}$ = 0.124	0.149	2.980	-0.238	I
		$\hat{\beta}$ = 0.381	0.748	4.489	0.907	IV
	0.5	$\hat{\alpha}$ = 0.123	-0.162	3.786	-0.058	I
		$\hat{\beta}$ = 0.382	0.725	4.356	1.198	VI

Table (6)

Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution α = 1/2, β = 9/8

N	P	Estimate	γ_3	γ_4	K	Dist^n
10	0.8	$\hat{\alpha}$ = 0.556	0.466	3.874	1.111	VI
		$\hat{\beta}$ = 1.307	2.538	15.684	0.226	IV
	0.7	$\hat{\alpha}$ = 0.560	0.310	3.954	0.258	IV
		$\hat{\beta}$ = 1.318	2.881	19.902	0.212	IV
	0.5	$\hat{\alpha}$ = 0.678	14.866	297.264	0.538	IV
		$\hat{\beta}$ = 1.568	18.611	401.156	0.663	IV
20	0.8	$\hat{\alpha}$ = 0.534	0.349	3.450	-1.938	I
		$\hat{\beta}$ = 1.214	1.105	4.974	1.674	VI
	0.7	$\hat{\alpha}$ = 0.538	0.187	3.321	1.857	VI
		$\hat{\beta}$ = 1.219	1.443	7.023	0.410	IV
	0.5	$\hat{\alpha}$ = 0.574	0.900	8.606	0.112	IV
		$\hat{\beta}$ = 1.242	2.820	23.313	0.179	IV
30	0.8	$\hat{\alpha}$ = 0.519	0.232	3.305	-2.137	I
		$\hat{\beta}$ = 1.176	0.782	3.838	-1.050	I
	0.7	$\hat{\alpha}$ = 0.521	0.075	3.243	0.219	IV
		$\hat{\beta}$ = 1.178	0.826	4.099	-2.679	I
	0.5	$\hat{\alpha}$ = 0.526	0.068	3.523	0.062	IV
		$\hat{\beta}$ = 1.183	1.001	4.849	1.356	VI

Table (6)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution α = 1/2, β = 9/8

N	P	Estimate	γ3	γ4	K	Dist^n
50	0.8	$\hat{\alpha}$ = 0.517	0.258	3.283	-0.988	I
		$\hat{\beta}$ = 1.159	0.620	3.685	-1.097	I
	0.7	$\hat{\alpha}$ = 0.517	0.185	2.987	-0.251	I
		$\hat{\beta}$ = 1.16	0.637	3.694	-1.063	I
	0.5	$\hat{\alpha}$ = 0.518	0.189	3.359	0.984	IV
		$\hat{\beta}$ = 1.163	0.783	4.033	-2.482	I
100	0.8	$\hat{\alpha}$ = 0.507	0.123	3.007	-0.268	I
		$\hat{\beta}$ = 1.143	0.591	4.053	1.526	VI
	0.7	$\hat{\alpha}$ =0.504	0.172	3.005	-0.267	I
		$\hat{\beta}$ = 1.142	0.691	4.399	0.838	IV
	0.5	$\hat{\alpha}$ = 0.509	0.147	2.909	-0.184	I
		$\hat{\beta}$ = 1.145	0.726	4.223	2.412	VI

Table (7)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution α = 9/8, β = 9/8

N	P	Estimate	γ_3	γ_4	K	Dist^n
10	0.8	$\hat{\alpha}$ = 1.306	0.925	5.809	0.308	IV
		$\hat{\beta}$ = 1.317	1.360	5.923	0.782	IV
	0.7	$\hat{\alpha}$ = 1.337	0.830	5.027	0.485	IV
		$\hat{\beta}$ = 1.343	1.760	7.619	0.498	IV
	0.5	$\hat{\alpha}$ = 1.535	4.192	42.042	0.209	IV
		$\hat{\beta}$ = 1.452	5.664	60.876	0.258	IV
20	0.8	$\hat{\alpha}$ = 1.225	0.710	4.709	0.490	IV
		$\hat{\beta}$ = 1.221	1.758	12.012	0.182	IV
	0.7	$\hat{\alpha}$ = 1.233	0.472	3.750	4.732	VI
		$\hat{\beta}$ = 1.227	1.770	10.898	0.213	IV
	0.5	$\hat{\alpha}$ = 1.288	13.467	317.16	0.478	IV
		$\hat{\beta}$ = 1.260	17.942	459.475	0.626	IV
30	0.8	$\hat{\alpha}$ = 1.176	0.333	3.436	-2.146	I
		$\hat{\beta}$ = 1.169	0.835	4.233	-19.235	I
	0.7	$\hat{\alpha}$ = 1.182	0.152	3.195	-1.828	I
		$\hat{\beta}$ = 1.172	0.729	3.612	-0.675	I
	0.5	$\hat{\alpha}$ = 1.226	0.324	3.854	0.357	IV
		$\hat{\beta}$ = 1.191	1.016	4.436	-5.421	I

Table (7)

Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution α = 9/8, β = 9/8

N	P	Estimate	γ_3	γ_4	K	Dist^n
50	0.8	$\hat{\alpha}$ = 1.146	0.378	3.317	-0.623	I
		$\hat{\beta}$ = 1.153	0.591	3.631	-0.998	I
	0.7	$\hat{\alpha}$ = 1.143	0.263	3.237	-0.669	I
		$\hat{\beta}$ = 1.153	0.626	3.646	-0.926	I
	0.5	$\hat{\alpha}$ = 1.165	0.408	3.884	0.623	IV
		$\hat{\beta}$ = 1.163	0.886	4.768	0.928	IV
100	0.8	$\hat{\alpha}$ = 1.134	0.053	3.194	0.178	IV
		$\hat{\beta}$ = 1.136	0.342	3.352	-0.868	I
	0.7	$\hat{\alpha}$ = 1.138	0.072	3.08	-0.983	I
		$\hat{\beta}$ = 1.138	0.282	3.062	-0.315	I
	0.5	$\hat{\alpha}$ = 1.145	0.084	3.169	0.765	IV
		$\hat{\beta}$ = 1.142	0.475	3.217	-0.405	I

Table (8)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution $\alpha = 9/8$, $\beta = 1/2$

N	P	Estimate	γ_3	γ_4	K	Dist^n
10	0.8	$\hat{\alpha}$ = 1.279	0.865	5.523	0.329	IV
		$\hat{\beta}$ = 0.576	1.565	7.757	0.357	IV
	0.7	$\hat{\alpha}$ = 1.280	-0.909	17.079	-0.042	I
		$\hat{\beta}$ = 0.581	2.301	14.523	0.214	IV
	0.5	$\hat{\alpha}$ = 1.450	2.554	29.809	0.134	IV
		$\hat{\beta}$ = 0.687	21.013	459.086	0.745	IV
20	0.8	$\hat{\alpha}$ = 1.181	0.380	3.249	-0.488	I
		$\hat{\beta}$ = 0.535	1.140	5.012	1.822	VI
	0.7	$\hat{\alpha}$ = 1.184	0.338	3.318	-0.726	I
		$\hat{\beta}$ = 0.536	1.294	5.926	0.661	IV
	0.5	$\hat{\alpha}$ = 1.199	-0.397	8.682	-0.030	I
		$\hat{\beta}$ = 0.541	1.943	13.434	0.182	IV
30	0.8	$\hat{\alpha}$ = 1.182	0.195	3.097	-0.392	I
		$\hat{\beta}$ = 0.522	0.794	3.942	-1.432	I
	0.7	$\hat{\alpha}$ = 1.182	0.163	3.007	-0.268	I
		$\hat{\beta}$ = 0.522	0.876	4.203	-3.584	I
	0.5	$\hat{\alpha}$ = 1.182	0.153	3.106	-0.480	I
		$\hat{\beta}$ = 0.524	1.256	6.003	0.562	IV

Table (8)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution α = 9/8, β = 1/2

N	P	Estimate	γ_3	γ_4	K	Dist^n
50	0.8	$\hat{\alpha}$ = 1.150	0.132	3.213	3.493	VI
		$\hat{\beta}$ = 0.515	0.658	3.795	-1.494	I
	0.7	$\hat{\alpha}$ = 1.15	0.042	2.939	-0.129	I
		$\hat{\beta}$ = 0.516	0.699	3.864	-1.676	I
	0.5	$\hat{\alpha}$ = 1.174	0.254	3.088	-0.346	I
		$\hat{\beta}$ = 0.519	0.851	4.247	-12.932	I
100	0.8	$\hat{\alpha}$ = 1.146	0.072	2.874	-0.118	I
		$\hat{\beta}$ = 0.509	0.610	4.326	0.643	IV
	0.7	$\hat{\alpha}$ = 1.144	0.045	2.890	-0.097	I
		$\hat{\beta}$ = 0.509	0.663	4.600	0.483	IV
	0.5	$\hat{\alpha}$ = 1.154	0.080	3.091	-1.043	I
		$\hat{\beta}$ = 0.510	0.748	4.545	0.790	IV

Table (5)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution α = 1/8, β = 3/8

n	P	Estimate	γ_3	γ_4	K	Dist[n]
10	0.8	$\hat{\alpha}$ = 0.129	0.802	11.158	0.068	IV
		$\hat{\beta}$ = 0.455	4.161	32.652	0.239	IV
	0.7	$\hat{\alpha}$ = 0.103	-0.586	6.158	-0.058	I
		$\hat{\beta}$ = 0.453	4.027	32.599	0.229	IV
	0.5	$\hat{\alpha}$ = 0.073	6.634	112.665	0.258	IV
		$\hat{\beta}$ = 0.534	27.418	810.437	0.937	IV
20	0.8	$\hat{\alpha}$ = 0.129	1.092	10.412	0.111	IV
		$\hat{\beta}$ = 0.410	1.579	7.474	0.409	IV
	0.7	$\hat{\alpha}$ = 0.121	-0.540	6.937	-0.048	I
		$\hat{\beta}$ = 0.411	1.620	7.803	0.378	IV
	0.5	$\hat{\alpha}$ = 0.126	-2.052	22.900	-0.077	I
		$\hat{\beta}$ = 0.411	2.261	16.633	0.178	IV
30	0.8	$\hat{\alpha}$ = 0.129	0.511	4.334	0.383	IV
		$\hat{\beta}$ = 0.394	0.952	4.278	-2.956	I
	0.7	$\hat{\alpha}$ = 0.129	-0.033	3.889	-0.013	I
		$\hat{\beta}$ = 0.394	0.962	4.273	-2.632	I
	0.5	$\hat{\alpha}$ = 0.123	-0.276	3.465	-0.112	I
		$\hat{\beta}$ = 0.394	0.957	4.396	-11.345	I
50	0.8	$\hat{\alpha}$ = 0.128	0.333	3.281	-0.620	I
		$\hat{\beta}$ = 0.388	0.723	4.009	-4.281	I
	0.7	$\hat{\alpha}$ = 0.127	0.156	3.212	-2.908	I
		$\hat{\beta}$ = 0.388	0.722	3.992	-3.509	I
	0.5	$\hat{\alpha}$ = 0.120	-0.115	3.705	-0.049	I
		$\hat{\beta}$ = 0.387	0.756	4.027	-3.131	I
100	0.8	$\hat{\alpha}$ = 0.126	0.206	3.383	1.107	VI
		$\hat{\beta}$ = 0.381	0.768	4.574	0.814	IV
	0.7	$\hat{\alpha}$ = 0.124	0.149	2.980	-0.238	I
		$\hat{\beta}$ = 0.381	0.748	4.489	0.907	IV
	0.5	$\hat{\alpha}$ = 0.123	-0.162	3.786	-0.058	I
		$\hat{\beta}$ = 0.382	0.725	4.356	1.198	VI

Table (6)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution $\alpha = 1/2$, $\beta = 9/8$

n	P	Estimate	γ_3	γ_4	K	Dist^n
10	0.8	$\hat{\alpha} = 0.556$	0.466	3.874	1.111	VI
		$\hat{\beta} = 1.307$	2.538	15.684	0.226	IV
	0.7	$\hat{\alpha} = 0.560$	0.310	3.954	0.258	IV
		$\hat{\beta} = 1.318$	2.881	19.902	0.212	IV
	0.5	$\hat{\alpha} = 0.678$	14.866	297.264	0.538	IV
		$\hat{\beta} = 1.568$	18.611	401.156	0.663	IV
20	0.8	$\hat{\alpha} = 0.534$	0.349	3.450	-1.938	I
		$\hat{\beta} = 1.214$	1.105	4.974	1.674	VI
	0.7	$\hat{\alpha} = 0.538$	0.187	3.321	1.857	VI
		$\hat{\beta} = 1.219$	1.443	7.023	0.410	IV
	0.5	$\hat{\alpha} = 0.574$	0.900	8.606	0.112	IV
		$\hat{\beta} = 1.242$	2.820	23.313	0.179	IV
30	0.8	$\hat{\alpha} = 0.519$	0.232	3.305	-2.137	I
		$\hat{\beta} = 1.176$	0.782	3.838	-1.050	I
	0.7	$\hat{\alpha} = 0.521$	0.075	3.243	0.219	IV
		$\hat{\beta} = 1.178$	0.826	4.099	-2.679	I
	0.5	$\hat{\alpha} = 0.526$	0.068	3.523	0.062	IV
		$\hat{\beta} = 1.183$	1.001	4.849	1.356	VI
50	0.8	$\hat{\alpha} = 0.517$	0.258	3.283	-0.988	I
		$\hat{\beta} = 1.159$	0.620	3.685	-1.097	I
	0.7	$\hat{\alpha} = 0.517$	0.185	2.987	-0.251	I
		$\hat{\beta} = 1.16$	0.637	3.694	-1.063	I
	0.5	$\hat{\alpha} = 0.518$	0.189	3.359	0.984	IV
		$\hat{\beta} = 1.163$	0.783	4.033	-2.482	I
100	0.8	$\hat{\alpha} = 0.507$	0.123	3.007	-0.268	I
		$\hat{\beta} = 1.143$	0.591	4.053	1.526	VI
	0.7	$\hat{\alpha} =0.504$	0.172	3.005	-0.267	I
		$\hat{\beta} = 1.142$	0.691	4.399	0.838	IV
	0.5	$\hat{\alpha} = 0.509$	0.147	2.909	-0.184	I
		$\hat{\beta} = 1.145$	0.726	4.223	2.412	VI

Table (7)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution $\alpha = 9/8$, $\beta = 9/8$

N	P	Estimate	γ_3	γ_4	K	Dist^n
10	0.8	$\hat{\alpha}$ = 1.306	0.925	5.809	0.308	IV
		$\hat{\beta}$ = 1.317	1.360	5.923	0.782	IV
	0.7	$\hat{\alpha}$ = 1.337	0.830	5.027	0.485	IV
		$\hat{\beta}$ = 1.343	1.760	7.619	0.498	IV
	0.5	$\hat{\alpha}$ = 1.535	4.192	42.042	0.209	IV
		$\hat{\beta}$ = 1.452	5.664	60.876	0.258	IV
20	0.8	$\hat{\alpha}$ = 1.225	0.710	4.709	0.490	IV
		$\hat{\beta}$ = 1.221	1.758	12.012	0.182	IV
	0.7	$\hat{\alpha}$ = 1.233	0.472	3.750	4.732	VI
		$\hat{\beta}$ = 1.227	1.770	10.898	0.213	IV
	0.5	$\hat{\alpha}$ = 1.288	13.467	317.16	0.478	IV
		$\hat{\beta}$ = 1.260	17.942	459.475	0.626	IV
30	0.8	$\hat{\alpha}$ = 1.176	0.333	3.436	-2.146	I
		$\hat{\beta}$ = 1.169	0.835	4.233	-19.235	I
	0.7	$\hat{\alpha}$ = 1.182	0.152	3.195	-1.828	I
		$\hat{\beta}$ = 1.172	0.729	3.612	-0.675	I
	0.5	$\hat{\alpha}$ = 1.226	0.324	3.854	0.357	IV
		$\hat{\beta}$ = 1.191	1.016	4.436	-5.421	I
50	0.8	$\hat{\alpha}$ = 1.146	0.378	3.317	-0.623	I
		$\hat{\beta}$ = 1.153	0.591	3.631	-0.998	I
	0.7	$\hat{\alpha}$ = 1.143	0.263	3.237	-0.669	I
		$\hat{\beta}$ = 1.153	0.626	3.646	-0.926	I
	0.5	$\hat{\alpha}$ = 1.165	0.408	3.884	0.623	IV
		$\hat{\beta}$ = 1.163	0.886	4.768	0.928	IV
100	0.8	$\hat{\alpha}$ = 1.134	0.053	3.194	0.178	IV
		$\hat{\beta}$ = 1.136	0.342	3.352	-0.868	I
	0.7	$\hat{\alpha}$ = 1.138	0.072	3.08	-0.983	I
		$\hat{\beta}$ = 1.138	0.282	3.062	-0.315	I
	0.5	$\hat{\alpha}$ = 1.145	0.084	3.169	0.765	IV
		$\hat{\beta}$ = 1.142	0.475	3.217	-0.405	I

Table (8)
Sampling distribution of the MLE for Type I censored samples according to Pearson's Type of distribution $\alpha = 9/8$, $\beta = 1/2$

N	P	Estimate	γ_3	γ_4	K	Dist^n
10	0.8	$\hat{\alpha} = 1.279$	0.865	5.523	0.329	IV
		$\hat{\beta} = 0.576$	1.565	7.757	0.357	IV
	0.7	$\hat{\alpha} = 1.280$	-0.909	17.079	-0.042	I
		$\hat{\beta} = 0.581$	2.301	14.523	0.214	IV
	0.5	$\hat{\alpha} = 1.450$	2.554	29.809	0.134	IV
		$\hat{\beta} = 0.687$	21.013	459.086	0.745	IV
20	0.8	$\hat{\alpha} = 1.181$	0.380	3.249	-0.488	I
		$\hat{\beta} = 0.535$	1.140	5.012	1.822	VI
	0.7	$\hat{\alpha} = 1.184$	0.338	3.318	-0.726	I
		$\hat{\beta} = 0.536$	1.294	5.926	0.661	IV
	0.5	$\hat{\alpha} = 1.199$	-0.397	8.682	-0.030	I
		$\hat{\beta} = 0.541$	1.943	13.434	0.182	IV
30	0.8	$\hat{\alpha} = 1.182$	0.195	3.097	-0.392	I
		$\hat{\beta} = 0.522$	0.794	3.942	-1.432	I
	0.7	$\hat{\alpha} = 1.182$	0.163	3.007	-0.268	I
		$\hat{\beta} = 0.522$	0.876	4.203	-3.584	I
	0.5	$\hat{\alpha} = 1.182$	0.153	3.106	-0.480	I
		$\hat{\beta} = 0.524$	1.256	6.003	0.562	IV
50	0.8	$\hat{\alpha} = 1.150$	0.132	3.213	3.493	VI
		$\hat{\beta} = 0.515$	0.658	3.795	-1.494	I
	0.7	$\hat{\alpha} = 1.15$	0.042	2.939	-0.129	I
		$\hat{\beta} = 0.516$	0.699	3.864	-1.676	I
	0.5	$\hat{\alpha} = 1.174$	0.254	3.088	-0.346	I
		$\hat{\beta} = 0.519$	0.851	4.247	-12.932	I
100	0.8	$\hat{\alpha} = 1.146$	0.072	2.874	-0.118	I
		$\hat{\beta} = 0.509$	0.610	4.326	0.643	IV
	0.7	$\hat{\alpha} = 1.144$	0.045	2.890	-0.097	I
		$\hat{\beta} = 0.509$	0.663	4.600	0.483	IV
	0.5	$\hat{\alpha} = 1.154$	0.080	3.091	-1.043	I
		$\hat{\beta} = 0.510$	0.748	4.545	0.790	IV